Analysis of upper extremes in skewed records using a non-Gaussion second order model

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Outline

- Laplace integral model
  - motivation
  - introduction
- Example
  - comparison to Gaussian approach
- Outlook
- Conclusion

Motivation

"Geostatistical data often display non-Gaussian features, such as heavy tails or skewness (...) and observations that would be 'outliers' under a Gaussian sampling process."[1]

[1] M. Blanca Palacios & Mark F. J. Steel (2006), "Non-Gaussian Bayesian Geostatistical Modeling", *Journal of the American Statistical Association*, Vol. 101, Nr. 474

Introduction: LI model

- Laplace Integral:

$$X_t = \int_{\mathcal{X}} f(t - x) \, d\Lambda(x) \ ,$$

where:

$\Lambda(x)$	Laplace measure
$f(\cdot)$	kernel
$\mathcal{X}$	here: $\mathbb{R}$
X_t	has characteristic function $\Phi_{X_t}(f(\cdot), \ldots)$

Estimating the kernel

- For symmetric kernels:

$$\widehat{f}(x) = \sqrt{2\pi}\frac{\mathcal{F}^{-1}\sqrt{\widehat{R}(\omega)}}{\sqrt{\int_{-\infty}^{\infty}\widehat{R}(\omega)\,d\omega}} ,$$

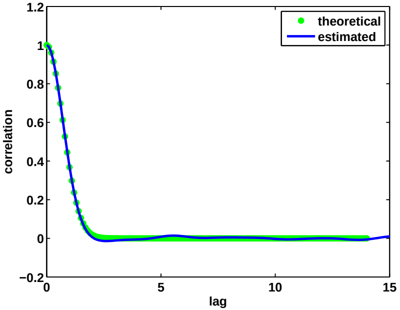
  where:

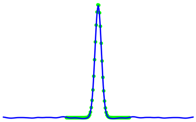
$\widehat{R}(\omega)$	estimate of spectrum
$\mathcal{F}^{-1}$	inverse Fourier transform

Estimate of correlationfunction

With previous normalisation:

$$\widehat{r}(\tau) = \widehat{f}(x) * \widehat{f}(x)$$



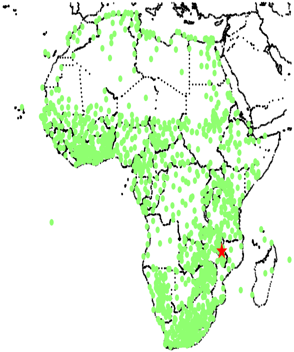


$$f(x) = \sqrt{2}\pi^{-1/4}e^{-2x^2}$$

Figure: Theoretical & estimated: correlation-function (left), kernel (right).

Example: Precipitation data

Observation stations in Africa:



Modeling recipe:

- estimate kernel
  $\rightarrow$ correlationfunction
- estimate parameters of generalised Laplace
- simulate X_t
- "plotting routine"

Example: Precipitation data

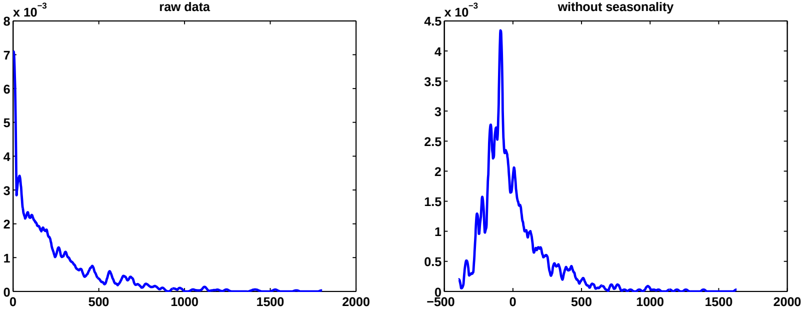


Figure: PDF of raw data (left) and after removing seasonality (right).

Example: Precipitation data

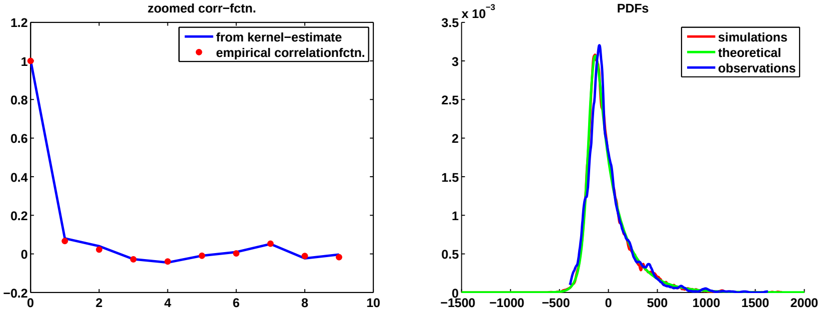


Figure: Correlation functions (left), PDFs after removing seasonality (right).

Note I

Asymmetric PDFs even through symmetric kernel.

Gaussian alternatives:

- Log-transformation
  - What about negative values, zero values?
  - How to handle seasonality?
- Box-Cox power transforms:
  - 1-2 extra parameters
  - "Continuous extension" for zero parameter values.

Needs more justification in spatial modeling.

Precipitation data - Gaussian approach

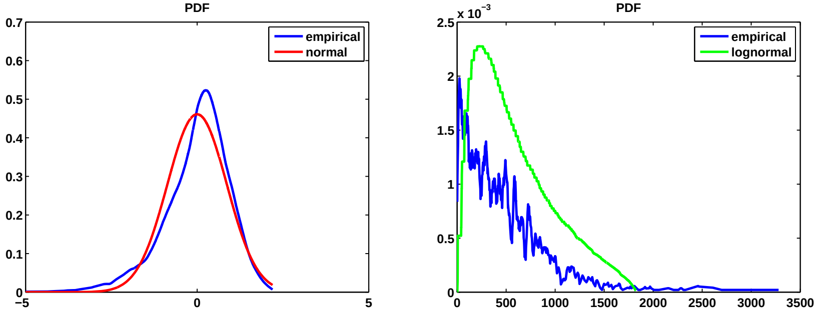


Figure: Seasons removed: PDFs of log-data (left) and empirical & lognormal PDF (right).

Precipitation data - Gaussian approach

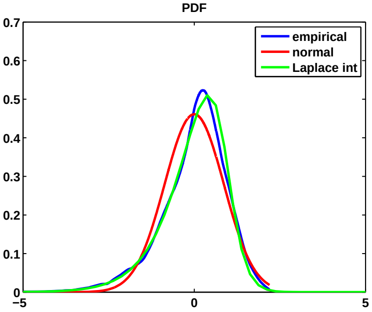


Figure: PDFs: log-data, Normal and LI-model fitted to log-data.

Future work

- incorporate asymmetric kernels. Such trajectories are not possible in Gaussian approach:

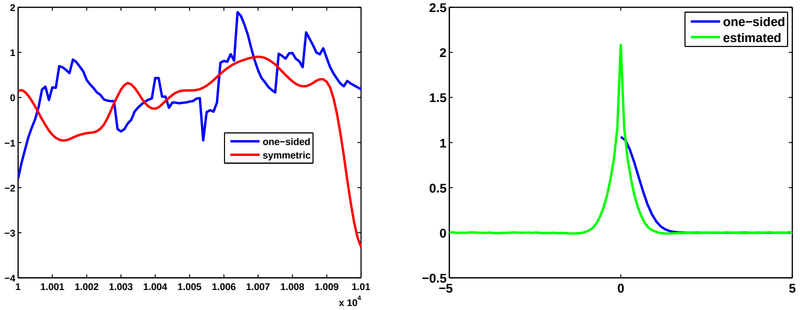


Figure: Symmetric & one-sided kernel: realisation (left), kernel (right).

- extend to spatial-temporal modeling

Conclusion

LI-model accounts for:

- heavier than Gaussian tails
- skewed data
- immediate way of modeling

Note II

... has Gaussian model as special case.