

List of required theorems

Part I

The page and theorem numbers below refer to my lecture notes from 2000.

OBS! Of course, the proofs of some of the results below make use of earlier results. In these cases, it suffices to state the earlier result in your proof.

p.1 : Aritmetikens fundamental sats.

p.5 : Prop. 4.

p.13 : CRT and Kor. 10.

p.15/16 : Fermat's lilla sats och Euler's sats.

p.16 : Sats 12, Prop. 13.

p.17 : Prop. 14, Sats 15.

p.21 : Möbius inversion formel, Sats 16.

p.33 : Gauß' lemma and reciprocity law.

p.37 : Prop. 23.

p.38 : Sats.

p.39 : Prop. 24.

p.41 : Theorem 26, Dirichlet's theorem (know the proof as far as the reduction to proving that $L(1, \chi) \neq 0$, i.e.: from the middle of p.42 (you may assume Theorem 26) to the middle of p.44).

p.45 : Theorem 31.

p.56 : Theorem 37 (ii), (iii).

p.58 : Sats 42.

p.59 : Kor. 43.

p.63 : Theorem 47.

p.68 : Prop. 48 (in the proof you may quote Thm. 47).

p.69 : Sats 50.

p.71 : Sats 51 (you may quote Sats 42, but must prove Sats 50).

p.75 : Theorem 55.

p.128 : Dirichlets approximationsats.

p.129 : Sats 103.

p.131 : Sats 104. OBS! I did not actually prove Sats 104, but rather the following strengthening of Sats 103, for which the method of proof is the same as that for Sats 104 :

Let $d > 0$ be an integer which is not a perfect square. Then there are

infinitely many integer pairs (x, y) solving the equation $x^2 - dy^2 = 1 \dots ()$.
We have a 1-1 correspondence between such pairs and real numbers given by*

$$(x, y) \leftrightarrow x + y\sqrt{d}.$$

Under this correspondence, the solutions to $()$ form a subgroup of the multiplicative group \mathbf{R}^\times of non-zero real numbers. This group is isomorphic to $\{\pm 1\} \times \mathbf{Z}$, in other words, there exists a fundamental solution (x_0, y_0) such that all solutions (x, y) are given explicitly by*

$$x + y\sqrt{d} = \pm(x_0 + y_0\sqrt{d})^n, \quad n \in \mathbf{Z}.$$

Part II

The page and theorem numbers refer to my lecture notes from 2002.

p.1 : Theorem 4.2.

p.3 : Theorem 5.1 (you may assume that the FTA holds in $\mathbf{Z}[\sqrt{-2}]$).

p.5 : Theorem 6.2.

p.6 : Prop. 6.3.