MATEMATIK   Dag : 070830   Tid : 8.30 - 13.30.
Göteborgs Universitet   Hjälpmedel : Inga
Peter Hegarty   Vakt : Peter Hegarty 0733-428321.

Tentamenskriving i Talteori (MAN 640)

$\geq$ 12.5 poäng, inkl. inlämningsuppgifterna, ger godkännt.

1 (3p) Does the congruence

$$3x^2 + 11x + 9 \equiv 0 \ (\mathrm{mod} \ 1003)$$

have any solutions ? (Motivate your answer !)

2 (3p) Prove the case $n = 4$ of Fermat's theorem.
    (OBS! You may use results on Pythagorean triples without proof).

3 (3p) As you all know, a real number whose decimal expansion either terminates or recurs must be rational. What can one say of a real number whose continued fraction expansion either terminates or recurs ? (Motivate your answer !)

4 (3p) State and prove Lagrange's theorem on sums of squares.
    (OBS! If you choose to prove the result using Minkowski's theorem, then you don't need to prove the latter).

5 (1.5p+1.5p) (i) Prove that $H(-44) = 4$ and write down all reduced positive-definite binary quadratic forms of this discriminant.
(ii) Give a variable substitution which converts the form

$$113x^2 + 42xy + 4y^2$$

to a reduced form (OBS! the form has discriminant -44).

6 (3p) State and prove Liouville's theorem on Diophantine approximation of algebraic numbers.

7 (2p+2p) (i) State and prove a formula for the Riemann zeta-function as an infinite product over the primes, indicating the range of its validity.

(ii) Using this, or otherwise, prove that the sum of the reciprocals of the primes diverges.

8 (0.5p+2.5p) (i) Define the Van der Waerden numbers $W(k, m)$.
(ii) Prove that

$$W(k, m) \geq \sqrt{2(k-1)} \, m^{\frac{k-1}{2}}.$$

(HINT : Consider a random m-coloring of $\{1, ..., n\}$.)

Obs! Tentan beräknas vara färdigrättad den 4 september. Då kan den hämtas i mottagningsrummet mellan kl. 12:30-13:00. Tentamensresultat lämnas också ut per telefon 772 35 09 *efter* kl. 14:00.