Wallenberg Academy programmes for mathematics 2014

The Knut and Alice Wallenberg Foundation, in cooperation with the Royal Swedish Academy of Sciences, will support 84 prominent researchers in Mathematics during 2014-2022. The funding amounts to a total of SEK 160 million, and in the first application round 15 positions have been filled.

Five established researchers from abroad will become visiting professors at Swedish universities:


Photo: Max Planck Institute for Gravitational Physics

Secrets of the universe revealed by Einstein's equations

Lars Andersson is at present Professor at Max Planck Institute for Gravitational Physics (Albert Einstein Institute) in Potsdam, Germany. He will be Visiting Professor at the Department of mathematics, KTH Royal Institute of Technology, Stockholm.

Despite the fact that Einstein's general theory of relativity is almost 100 years old, it is not fully understood yet. The theory implies a description of the development of the universe, as well as individual events in space like black holes. However, the models, which are currently used by physicists, describe the physical reality only under limited, special assumptions. Solutions of Einstein's equations under less restrictive assumptions are required in order to better understand the physical reality.

One way to look at the theory is to formulate Einstein's equations as a so-called initial value problem. This approach means that if we can describe the state of (for example) the universe at a given moment, the equations will fully predict its evolution over time.

Simple examples of such solutions can be found in classical mechanics. If you throw a ball, knowing the ball's initial position and speed, as well the strength of the gravitational field and the air resistance, you can calculate the ball's trajectory.

The initial conditions for Einstein's equations must satisfy the so-called constraint equations. Solutions for Einstein's equations can only be found after having solved the constraint equations. At present, they can be solved for short time periods but not for longer ones. One of the goals of the project is to find more solutions of the constraint equations and describe their properties. Studying exotic phenomena, the so-called singularities, where the laws of physics don't hold, is another goal. The most well-known such singularity is the Big Bang – the beginning of the universe.


Photo: University of Insubria

Better tools for computer simulations

Stefano Serra Capizzano is at present Professor at University of Insubria, Italy. He will be Visiting Professor at the Computational Group, Department of Information Technology, Uppsala University.

At present, we would like to understand the behavior of complex systems such as the global earth climate, osteoporosis, the human brain, or the geological changes of the earth's surface altitude. It is not possible to analyze such complex processes solely with paper and pencil, as they do not have analytic solutions. Researchers build sophisticated models with the help of ever more powerful computers. In order to ascertain that such computer simulations yield sufficiently accurate results, it is required to develop rigorous analytic tools and mathematical proofs. Only then it can be assumed that the simulations correspond to the real phenomena.

The Computational Group and geologists from Uppsala University have worked together on applying computer simulation to study historical changes in Earth's surface altitude under the influence of geological periods of glaciation and deglaciation. Modeling Earth's altitude is not a simple exercise. The underlying physical processes are complex, the time and the space scales are enormous, the interior of the Earth is heterogeneous, and the thickness of the ice variable.

The initial, somewhat simplified models have already shown that the classical mathematical descriptions are not useful. The group is developing new methods in order to ascertain better accuracy of numerical solutions and shorter computing time for the simulations.

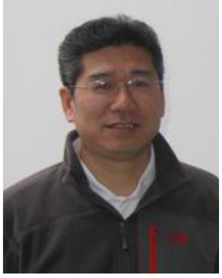
Photo: Mathematisches Forschungsinstitut Oberwolfach/Renate Schmid

Finding patterns in chaos

Ai-Hua Fan is at present Professor at University of Picardie Jules Verne in Amiens, France. He will be Visiting Professor at the Centre for Mathematical Sciences, Lund University.

One of the subjects studied by the theory of dynamical systems is the exploration of mathematical models of chaotic phenomena, i.e. situations in which small changes in the initial conditions cause large deviations in the evolution of the system. This property makes it impossible, for example, to make long-term weather predictions, since weather is a chaotic phenomenon. Some degree of order exists in such systems though. We know with a high degree of confidence that a snowfall in Sweden is highly unlikely in July.

The study of regular patterns in chaotic systems is called ergodic theory. It was introduced over a hundred years ago by an Austrian physicist, Ludwig Boltzmann, who laid the foundations of statistical mechanics while studying the motion of gas particles.

Subsequently, the ergodic theory has been extended to the study of long-term evolution of ergodic averages in dynamical systems. For example, mathematicians were able to show that for a certain type of chaotic systems the initial conditions had no bearing on the development far into the future – the system "forgets" its past.

The methods needed to solve this type of problems are found in the intersection of many areas of mathematics, including harmonic analysis, probability theory, statistical mechanics, number theory, and the theory of fractals. The research group in Lund hopes to benefit from professor Fan's broad mathematical expertise in order to make further progress in its studies of ergodic theory.

Photo: Loughborough University

Mathematics of solitary waves

Mark David Groves is at present Professor at Saarland University in Saarbrücken, Germany. He will be Visiting Professor at the Centre for Mathematical Sciences, Lund University.

Even though water waves are some of the most common examples of wave motion, their mathematical interpretation is surprisingly difficult. The problem stems from the high degree of non-linearity of equations, which describe water waves. Therefore, researchers have been able to describe only certain aspects of such a motion in their simplified models.

The proposed project will develop a more complete model with focus on deeper understanding of solitary waves. Solitary waves can travel long distances with a constant shape and velocity. A British engineer, John Scott Russell, discovered such a wave first in 1834. He followed it for several kilometers along the Union canal between Glasgow and Edinburgh. Mathematical descriptions of such waves have found many applications, for example, in fiber optics and information technology.

While the wave observed by Russell was two-dimensional, since it exhibited no movement in the direction perpendicular to the direction of its progress, a theory of three-dimensional solitary waves has been initiated over the last ten years.

However, so far the progress of the theory has been limited by the physically unreasonable assumption of strong surface tension relative to the water's depth. The project will study three-dimensional solitary waves in the presence of weak surface tension. The participation of Mark Groves, a world expert on non-linear mathematics of water waves, may make it possible to reach higher level of understanding of three-dimensional water waves.

Photo: Universität Bern

Looking for order in absence of symmetry

Christiane Tretter is at present Professor at the University of Bern, Switzerland. She will be Visiting Professor at the Department of Mathematics, Stockholm University.

Mathematical models for transport in nano-structures and other microscopic systems have flourished recently. They have many applications in fiber optics, quantum conductors, and even some processes in the human body.

Most of the mathematical models developed to date assume symmetry. However, many applications, including electricity, theory of elasticity, or magneto-hydrodynamics lack symmetry. Those systems are very sensitive and a small perturbation can make the time evolution of the system difficult to predict.

This implies that numerical models are unreliable. Therefore it is desirable to develop rigorous analytic understanding of problems describing non-symmetric systems.

The goal of the joint research project is to develop a theory of non-symmetric systems, which lies within Christiane Tretter's area of expertise. By combining methods involving several branches of mathematics – theory of differential operators, analysis, complex analysis, and geometry – one may take further steps in response to the challenges of mathematical physics.

Sex established researchers will receive funding for a post-doctoral position in Sweden open to researchers from abroad:

Photo: KTH Royal Institute of Technology

Geometric objects contribute to deeper understanding of mathematical calculations

Mats Boij will receive funding for a postdoctoral position for international researchers at his group at the Department of mathematics, KTH Royal Institute of Technology, Stockholm.

The group at KTH studies a broad spectrum of problems in algebraic geometry, which is the desired specialization of the candidate for the post-doctoral position. Some of those problems intersect other fields of mathematics as well. Algebraic geometry has been successfully applied to cryptography, game theory in economics, and string theory in theoretical physics. There have been some industrial applications in computers and airplane construction as well as in robotics.

Algebraic geometry is the study of solutions of polynomial equations. It originated a long time ago in ancient Greece. During medieval times, Arab and Persian mathematicians were interested in such problems. Rene Descartes' introduction of the coordinate system in the 17th century was a significant turning point. It allowed linking algebraic equations with geometric objects defined by such equations. Some gradual progress in the understanding of such geometric constructions followed, but only in the 20th century research on algebraic geometry picked up pace. The development of abstract mathematical concepts contributed to solving many old problems.

Computers have been used to perform extensive calculations over the last 50 years. However, even today's powerful computers have limitations to their speed, and some calculations for solving seemingly simple problems may take longer than a lifetime.

Photo: KTH Royal Institute of Technology/ Ann-Britt Öhman

Understanding chaos in the quantum micro-world

Pär Kurlberg will receive funding for a postdoctoral position for international researchers at his group at the Department of mathematics, KTH Royal Institute of Technology, Stockholm.

The post-doctoral position is designed for a candidate with expertise in number theory. Eventually, research in quantum chaos can lead to applications in microelectronics, and therefore it may be of interest to computer and communications industries.

In a chaotic system an initial perturbation can be significantly amplified over time. Therefore it is not possible to predict the future behavior of such a system. One cannot, for example, possible to rule out that the earth will be ejected from the solar system long before the sun is extinguished.

It is even more difficult to understand chaotic phenomena in the micro-world governed by the laws of quantum mechanics. For example, it is not possible to determine exact trajectories of the particles. This limitation is required by the Heisenberg principle. Thus quantum mechanics defies one of the fundamental hallmarks of chaos: tightly convoluted but divergent trajectories.

However, the world is built up from microscopic phenomena. How can macroscopic chaos arise in a universe whose building blocks are governed by the laws of quantum mechanics? This question has been hotly debated. From the point of view of mathematical physics the problem can be formulated as the following question: in what way is the classical chaos expressed in the properties of the corresponding quantum mechanical system? This question gives rise to problems, which can be approached by using methods developed in number theory.

Developing an abstract theory of a theory

Volodymyr Mazorchuk will receive funding for a postdoctoral position for international researchers at his group at the Department of Mathematics, Uppsala University.

A common way to solve a mathematical problem is to first study its simplification. However, it often leads to a situation in which some useful information is lost. In order to develop a more sophisticated theory you often need to go back and transform the simplified problem into a more complex one.

This process is at work in the category theory, a branch of modern mathematics developed after the Second World War. The theory translates large areas of mathematics into a more general setting. Such upgrading of concepts from one area of mathematics to more general concepts is called categorification.

Reformulating mathematics into a more abstract setting can reveal a common structure in several seemingly unrelated fields of study. For example, categorification of the so-called Khovanov homology led to the development of a new theory, called higher representation theory.

Khovanov homology was developed in order to study ways to distinguish different knots. A knot can be imagined as a string with its ends melded together. Knot theory, an active area of research, finds its applications, among others, in the theory of low-dimensional spaces. One of the goals of the Uppsala group's research is to generalize the higher representation theory, and thus lay foundations to its abstract version.


Photo: Chalmers University of Technology

New ways of looking at Dirac equations

Andreas Rosén will receive funding for a postdoctoral position for international researchers at the Harmonic Analysis and Partial Differential Equations Group at the Department of Mathematical Science, Chalmers University of Technology and the University of Gothenburg.

Beginning with Newton in the 17th century, many mathematical applications start with a system of differential equations. The solutions can, for example, yield a weather forecast for a week-end, describe the flow of traffic, or growth of bacteria in a Petri dish.

Dirac equations, the subject of the proposed research, are a special category of differential equations. The British physicist and Nobel Prize laureate, Paul Dirac, formulated his equations in 1928 in order to obtain the quantum mechanical description of the motion of an electron. Dirac equations have played an important role in pure mathematics in the last fifty years.

In the 1960s, Michael Atiyah and Isadore Singer developed a new way of looking at some differential equations. Instead of looking for solutions, they asked the question about the size of the solution set. Their theory led to significant progress in the underlying mathematics.

The proposed project is going to utilize the developments in mathematics over the last fifty years. The goal is to study Dirac equations by using techniques of harmonic analysis. Harmonic analysis has been developed in order to study differential equations.


Photo: Jann Lipka

Computer analysis of movement patterns of fluid drops

Anna-Karin Tornberg will receive funding for a postdoctoral position for international researchers at the Numerical Analysis Group at the Department of Mathematics, KTH Royal Institute of Technology, Stockholm.

The new area of research – micro-fluidics, or microscopic fluid mechanics – has been developed to satisfy the need for understanding the manipulation of ever-smaller nano-objects. One emerging technology is “droplet micro-fluidics” in which very small droplets of water suspended in oil contain samples. Their surface is coated with an active substance, which keeps the droplets apart. Each droplet, less than a thirty thousandth millimeter in diameter, can contain a unique combination of substances or biological samples.

With the growing interest in micro-fluidics there is a need to better understand the underlying processes and their biological applications. The challenge is to develop accurate and fast methods for computer simulations of micro-fluids, particularly in three dimensions. One of the difficulties of such modeling stems from the variability of the water droplets, which tend to move around and undergo deformations. This is difficult to simulate with accurate and fast results.

The proposed research project will include the active substance coating of the droplets in the simulations, as well as develop new methods in order to circumvent certain difficulties. The goal is to develop accurate and fast methods for computer simulations for micro-fluid droplets coated with active substance.


Photo: Uppsala University

A step closer to a solution of an old problem

Warwick Tucker will receive funding for a postdoc position for international researchers at his group at the Department of Mathematics, Uppsala University.

An old problem from celestial mechanics, the so-called n-body problem, asked to describe the evolution of trajectories for any number of objects affected by gravitational interactions. The objects could be planets in the solar system, stars in a galaxy, or stars in a cluster.

The general solution, even only for three bodies, is not known. Attempts to solve the problem for three and more bodies gave rise to a new area of mathematics – the modern chaos theory.

A special solution of the n-body problem in which the bodies rotate in a plane around their common center of gravity is called relative equilibrium. The question if there is finitely many such relative equilibria has been intensely studied over the last ten years. The answer that in the case of four bodies the number is finite, and lies between 32 and 8,472, has been achieved recently.

The research group at Uppsala University hopes to find the exact number of relative equilibria in the case of four bodies and to ascertain that the number for five bodies is also finite. By applying modern numerical methods and powerful computers, the project is going to bring computational mathematics and pure mathematics closer together.

Four researchers holding Swedish doctorates will be recruited to post-doctoral positions at universities abroad and will receive support for two years upon returning to Sweden:


Photo: Gaudeamus, Stockholm University

Focusing on the special properties of mathematical objects

Per Alexandersson received his Ph.D. in mathematics at Stockholm University in 2013. He will hold a postdoctoral position in Professor James Haglund's research group at the University of Pennsylvania in Philadelphia, USA.

The research project to be undertaken together with Professor Haglund consists of two related problems in the representation theory and the theory of symmetric functions, i.e. functions, which do not change under permutations of their variables. Irreducible characters are the key objects studied in the theory of representations. They can be thought of as the primary indivisible elements from which larger objects are built. One can think of an analogy with prime numbers, which constitute indivisible factors of all whole numbers.

The study of the important concept of a bi-graded ring stands at the center of the first problem. Some of the key questions involve the properties of the ring. Some researchers think that by studying the geometric properties of the so-called hyper-plane arrangements one can find the sizes of the ring's components. However, the problem is still open. Several eminent mathematicians are active in this area of research.

The second problem consists of the study of the coefficients of certain symmetric polynomials. Their applications spread over several fields of mathematics including representation theory, harmonic analysis, and combinatorics. In addition to their significance in pure mathematics, symmetric functions and the representation theory have found many applications. For example, they are used in quantum physics, and in image compression.

Revealing properties of complex surfaces

Gabriel Bartolini received his Ph.D. in mathematics at Linköping University in 2012. He will hold a postdoctoral position in Professor Antonio F. Costa's research group at National Distance Education University (UNED) in Madrid, Spain.

Ever since Bernhard Riemann introduced the idea of a Riemann surface in his dissertation in 1851, it has become a central concept in many areas of mathematics. Riemann used it in his study of complex analysis, which is the theory of functions of complex numbers.

A complex number is a sum of a real number and an imaginary one, where the unit of an imaginary number is the square root of minus 1. Real numbers correspond to points of a straight line, and complex numbers to points of a two-dimensional plane in the Cartesian coordinate system.

One way to think about a Riemann surface is by imagining a deformed complex plane. An important property of a certain class of so-called compact Riemann surfaces is the number of holes they enclose. For example, a swim ring and a coffee cup with a handle have precisely one hole each, as opposed to a sphere, which doesn't have any.

Riemann showed that all compact Riemann surfaces with no holes are equivalent to a sphere. Riemann and other mathematicians were also able to show that all such surfaces with one hole can be described by using one complex parameter. However, it is significantly more difficult to describe compact Riemann surfaces with the number of holes higher than one. Gabriel Bartolini is planning to study geometric properties of Riemann surfaces and their symmetries together with Professor Antonio Costa and his research group in Madrid.

Telling apart a knot from a knot

Georgios Dimitroglou Rizell received his Ph.D. in mathematics at Uppsala University in 2013. He will hold a postdoctoral position in Professor Ivan Smith's research group at University of Cambridge, UK.

Symplectic manifolds are spaces of even (2, 4, 6, …) dimension in which one can define and study so-called Hamiltonian dynamical systems. The concept was originally developed in order to solve problems of classical mechanics like finding the trajectories of the planets in the solar system, or understanding the movement of a charged particle in a magnetic field. Symplectic manifolds studied presently can be viewed as generalizations of such dynamical systems.

Georgios Dimitroglou Rizell is interested in applying methods inspired by results in the theory of symplectic manifolds to knot theory. A knot can be imagined as a string whose two ends are melded together. There are infinitely many knots. One of the challenges of the modern knot theory, which has flourished during the last thirty years, is to find computationally feasible methods to tell knots apart.

Ivan Smith, the proposed host at Cambridge University, is currently working on such methods by extending ideas from the theory of symplectic manifolds to other settings. Researchers hope that such extensions will make progress in knot theory possible. Knots have applications in string theory of mathematical physics as well as in studies of DNA in biology.

Mathematics of earth's interior and chaos

Jens Wittsten received his Ph.D. in mathematics at Lund University in 2010. He will hold a visiting research position in Professor Setsuro Fujiie's group at Ritsumeikan University in Kyoto, Japan.

Jens Wittsten's area of expertise, microlocal analysis, has been developed in order to study solutions of partial differential equations. Its techniques have many applications.

For example, the theory has been used to study the accuracy of the representation of earth's interior based on earthquake data. The description cannot be complete because the problem is too complex and the equations used to describe it are non-linear and therefore difficult to analyze. In addition, there is not enough data from the measurements. One tries to find the best interpolation methods in order to get the best possible image.

New developments show that the theory can be applied also to dynamical systems, i.e. mathematical models of phenomena which evolve with time, like fluctuations of stock prices, or movements of particles in gas. So far microlocal analysis has been applied only to chaotic systems where noise is absent. The extension of the theory allowing for the presence of noise is the next step. Noise can cause large divergence in the evolution of a chaotic system.

The goal of the project is to find properties of the model, which are robust with respect to noise. Professor Fujiie and his group have expertise in this field and collaboration with them can prove to be very productive.