Mats Rudemo, March 31, 2003

Brief solutions for problems in Examination in Statistical Image Analysis, March 12, 2003

Most of the solutions given below are more brief than expected at the exam – the solutions given for the exam March 2001 are more representative of what is expected.

Problem 1.

Figure 1 (in the problem set) shows two images obtained from an experiment where ultrasound is used to locate inclusions in a steel specimen. Inclusions detected lie between 0.9 and 1.1 mm below the surface of the specimen. The steel has been rolled in a direction corresponding to the horizontal direction in the images. Inclusions are typically caused by small amounts of sulphur or oxygen and may start fatigue cracks when the steel is used in constructions and is subject to stress. Each white spot (blob) is supposed to correspond to one inclusion. (There are about 7 spots in the left image and about 9 spots in the right image.) For each specimen one has one image with about 9000 rows and 5000 columns and the images in Figure 1 are actually subimages of one such large image. Each pixel corresponds to an area of about $10^{-6}m \times 10^{-6}m$.

a) *Suggest a method for counting the number of spots in images such as those in Figure 1. Describe also how one can estimate a position for the centre (suitably defined) of each spot.*

One possibility is to smooth the the image by a Gaussian filter, to compute the histogram and to find a threshold from the histogram. If the histogram is bimodal the threshold may be chosen as the minimum between the two modes, and in a more complicated case by a mixture model with, for instance, two normal components.

After thresholding some morphological operations such as openings and thinnings may necessary to regularize the spots found.

Then count the number of spots.

Then estimate the spot centres as follows. Let C denote the set of pixels for one spot, and let n_C denote the number of pixels in the spot. Identify a pixel with its corresponding pixel centre (s,t), and define the spot centre $(\overline{s},\overline{t})$ by

$$\overline{s} = \frac{1}{n_C}\sum_{(s,t)\in C} s, \qquad \overline{t} = \frac{1}{n_C}\sum_{(s,t)\in C} t. \tag{1}$$

b) *One is also interested in estimating the size and shape of the spots. Suggest some method for doing this.*

The size of a spot (in pixel units) we estimate by the number of pixels in the spot.

Assuming that the spots are approximately elliptical with vertical and horizontal axes one could estimate the shape of a spot by the quotient of the height and the width of the spot.

c) *Suggest one method (or several methods) to test if the spots are placed in a purely random manner in the rectangles regarded. Looking at the images, do you expect deviations from a purely random placement of the spots?*

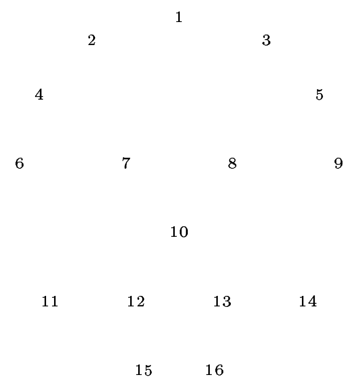
One possibility would be to compute an estimate of the K-function, compare the answer to Problem 2**a)** in the exam 2001 for details, to plot the square root of it and to compare with the corresponding function for a Poisson process to find clustering or inhibition.

However, the K-function is particularly sensitive to isotropic deviations from a Poisson process. And in our case it seems as the deviations are clusters in the horizontal (the rolling) direction. It is possible to construct tests that are more sensitive to this type of devaitions. One could, for instance, regard only the vertical spot coordinates ($\bar{t}$ in **a)** above) and plot the accumulated number of points as a function of the vertical coordinate. One would then expect to find more clustering than what corresponds to a Poisson process.

Problem 2. *Suppose that for 10 individuals we have 20 photos acquired at different occasions. All photos are supposed to be taken en-face (from forward) as shown in the left part of Figure 2 (in the problem set) where one photo for each of the persons is available. Suppose that we also have an image analysis programme that can identify 16 landmark points on a face as shown in the right part of Figure 2. The object is now to describe a method that together with a camera may be used for automatic discrimination between the 10 persons by use of suitable functions of the 16 landmarks. (For simplicity we only regard these 10 persons here.)*

a) *Suggest three size-dependent features that may be tried for discrimination, that is should be useful if the distance between the camera and the persons photographed is constant. Describe the computation of these features from the landmarks. Describe in words how you may implement the computations (without giving programmes) in a programme system like matlab.*

Draw a figure where you number the 16 landmark points, for instance as indicated here:



Let $z = (x, y)$ denote the coordinates of a point in 2D and let $|z - z'| = ((x - x')^2 + (y - y')^2)^{1/2}$ denote the distance between two points $z = (x, y)$ and $z' = (x', y')$. Let $Z_i, i = 1, \ldots, n$, $n = 16$, denote the 2D coordinates of the 16 landmarks. Choose, for instance, the three size-dependent features

$$X_1 = |Z_9 - Z_6|, \;\; X_2 = |Z_{13} - Z_{12}| \text{ and } X_3 = |Z_{10} - 0.5(Z_{15} + Z_{16})|. \quad (2)$$

(It could be a good idea not to rely too much on the landmarks $Z_1, \ldots, Z_5$ as they might be heavily affected by hair-cutting.)

b) *Suggest three size-independent features that may be tried for discrimination.*

Let A denote the area of the convex hull of the points $Z_6, \ldots, Z_{16}$ (or the convex hull of the points $Z_1, \ldots, Z_{16}$). Put $a = 1/\sqrt{A}$. Then

$$X_4 = \alpha|Z_9 - Z_6|, \;\; X_5 = \alpha|Z_{13} - Z_{12}| \text{ and } X_6 = \alpha|Z_{10} - 0.5(Z_{15} + Z_{16})| \quad (3)$$

are three possible size-independent features.

c) *Give a statistical model for the landmark data for the 10×20 images. How would you estimate parameters in the model?*

Let Y denote the 32-dimensional column vector formed by the coordinates of $Z_1, \ldots, Z_{16}$.

One possible statistical model is to assume that for person $i, i = 1, \ldots, 10$, the vector Y has a 32-dimensional normal distribution with mean μ_i and covariance matrix C_i. Perhaps one could assume that the covariance matrices are identical and equal to C.

To estimate parameters we let Y_{im}, denote the mth (32-dimensional) observation vector for the ith person. Put $n = 20$ and use the estimates

$$\hat{\mu}_i = \frac{1}{n} \sum_{m=1}^{n} Y_{im}, \quad i = 1, \ldots, 10. \tag{4}$$

If we make no assumption on equality of the covariance matrices we use the covariance matrix estimates

$$\hat{C}_i = \frac{1}{n-1} \sum_{m=1}^{n} (Y_{im} - \hat{\mu}_i)(Y_{im} - \hat{\mu}_i)^T, \quad i = 1, \ldots, 10. \tag{5}$$

If we assume equality of the covariance matrices we use instead the estimate

$$\hat{C} = \frac{1}{10}(\hat{C}_1 + \ldots + \hat{C}_{10}) \tag{6}$$

for the common covariance matrix C.

d) *Suggest a method for finding a suitable set of features for solving the discrimination problem with a low error rate.*

Let X_j, $j = 1, \ldots, J$, be possible feature variables that could be used. Examples of how these variables could be chosen are given in the solutions to **a)** and **b)**. Further, J could be a fairly large number, say of the order 50. One could choose the majority of these variables by looking at distances between groups of points in some systematic manner, but presumably it is better to use common sense in the selection of potential feature variables.

One could then use forward selection for choosing a suitable set of features, which in the present case takes the following form:

- Start by chosing the variable that considered alone gives the smallest error-rate estimate.
- Then choose a second variable that together with the first chosen variable gives the smallest error-rate estimate.
- Iterate this procedure of chosing an additional variable that together with the already chosen variables gives the smallest error-rate estimate, until the error rate estimate has reached a minimum

For the error-rate estimate we use the cross-validation error-rate estimate computed by successively leaving out one observation vector.