## PART 3 APPLICATONS

## 8 Analysis of microarray images

Read the following parts of Ekstrøm et al. (2004):
p 2270 The whole page
p 2271 The whole page
p 2272 The whole page
p 2273 Read until the text starting "A polynomial-hyperbolic spot shape family". Look at Fig. 3 to get an idea of how this spot shape looks.
p 2274 Read until the formula for $L_{2}$, that is disregard censored (saturated) pixels. Read also the last lines on RESULTS on that page
p 2275 The whole page
p 2276 Read until "Reconstruction of saturated pixels"

Read the following parts of Ekstrøm et al. (2005):
The Abstract
p 1 The whole page
p 2 The whole page
p 3 Skip the last paragraph starting "However, some pixels"
p 4 Read only the bottom lines with "Results"
p 5 The whole page
p 6 The whole page
p 7 Skip the last part with "Reconstruction of saturated values"
p 8 Regard only Figure 1

## 9 Two-dimensional electrophoresis

Read the following parts of Gustafsson, Blomberg \& Rudemo (2002):
Abstract

1. Introduction (skim this part)
2.1 Image warping
2.2 Gel images(skim this part)
2.3 Warping step I: current leakage correction (skip this part)
2.4 Warping step II: image alignement (skim this part)
2.5 Evaluation (skip this part)
2. Results (skim this part)
3. Discussion (skip this part)

## 10 Aerial photographs of forests

Read the following parts of Dralle \& Rudemo (1997):
Abstract
Introduction (skim this part)
Data (skim this part)
Problem specification
A model for the grey-level maxima given tree positions and heights
Parameter estimation
Results
Discussion
Conclusions

Read the following parts of Larsen \& Rudemo (1998):
Abstract

1. Introduction
2. The opticaL model (skim this part)
3. Local correlation maxima
4. Experiment
5. Discussion
6. Conclusions

## 11 Diffusion

### 11.1 Tracking a single diffusing particle

Let $X_{i}$ denote the position at time $i \Delta t, i=0,1, \ldots, K$, of a diffusing particle in $d$ dimensional space, where $d=1$, 2 or 3 in applications. We assume that

$$
\begin{equation*}
X_{i}=X_{i-1}+\Delta G_{i} \tag{86}
\end{equation*}
$$

where $\Delta G_{i}$ are independent $d$-dimensional normal vectors with a mean vector with all components zero and a covariance matrix

$$
\begin{equation*}
C\left(\Delta G_{i}\right)=2 D \Delta t I \tag{87}
\end{equation*}
$$

where $D$ is the diffusion coefficient and $I$ is the $d$-dimensional unit matrix. Thus in each dimension the diffusing particle has a normally distributed increment with mean zero and variance $2 D \Delta t$, and the increments in different dimensions and at different time-points are all independent.

Let $\|x\|$ denote the Euclidean norm in $d$-dimensional space, that is $\|x\|^{2}=\sum_{j} x_{j}^{2}$ if $x$ has components $x_{1}, \ldots, x_{d}$. Then

$$
\begin{equation*}
\mathbf{E}\left(\sum_{i=1}^{K}\left\|\Delta G_{i}\right\|^{2}\right)=2 d D \Delta t K \tag{88}
\end{equation*}
$$

and it follows that

$$
\begin{equation*}
\hat{D}=\frac{1}{2 d \Delta t K} \sum_{i=1}^{K}\left\|\Delta G_{i}\right\|^{2} \tag{89}
\end{equation*}
$$

is an unbiased estimate of the diffusion coefficient $D$.
We can also obtain a confidence interval for $D$ with, say, confidence degree $95 \%$. The variable

$$
\begin{equation*}
\chi^{2}=\frac{1}{2 D \Delta t} \sum_{i=1}^{K}\left\|\Delta G_{i}\right\|^{2} \tag{90}
\end{equation*}
$$

is chi-square distributed with $d K$ degrees of freedom. Thus

$$
\begin{equation*}
\operatorname{Pr}\left(\chi_{.025}^{2}<\chi^{2}<\chi_{.975}^{2}\right)=0.95 \tag{91}
\end{equation*}
$$

Straightforward computations give that (91) can be rewritten

$$
\begin{equation*}
\operatorname{Pr}\left(\frac{d K}{\chi_{.975}^{2}} \hat{D}<D<\frac{d K}{\chi_{.025}^{2}} \hat{D}\right)=0.95 \tag{92}
\end{equation*}
$$

and we see that

$$
\begin{equation*}
\frac{d K}{\chi_{.975}^{2}} \hat{D}<D<\frac{d K}{\chi_{.025}^{2}} \hat{D} \tag{93}
\end{equation*}
$$

is a confidence interval for $D$ with confidence degree $95 \%$.

### 11.2 A pixel-based likelihood framework for analysis of fluorescence recovery after photobleaching

Read the following parts of Jonasson et al. (2008):
Read Summary
Skim Introduction
In Theory:
Read Model
Skip Fluorescence intensity and fluorochrome concentration
Skip The detection point spread function
Skip Materials and methods, but read the last part starting with "To maximize the loglikelihood" on top of page 265
In Results read only the last part Diffusion in PEG solutions
Read Discussion, conclusions and outlook

### 11.3 Estimation of particle concentration from fluorescent particle counting

Read the following parts of Röding et al. (2011):
Read Abstract
Skim I. Introduction
Read II. Theory till A. Trajectory length distribution
Skim A. Trajectory length distribution
Skim B. Number concentration
Skim III. Simulation study
Skim IV. Experimental results
Read V. Discussion and conclusion
Skip Appendix A, B and C but have a look at Fig. 8

Read the following parts of Röding et al. (2013):
Read Summary
Skim Introduction
Read Theory and methods, Concentration measurements till equation (3) on p 21, skim the rest of this section till Bootstrap confidence intervals
Skip Bootstrap confidence intervals
Skim Simulation study
Skim Experimental results
Read Discussion and conclusion
Skip Appendix A-D

