

Platonism and Anti-Platonism in Mathematics

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This is a painful book to read. It is not that the author is full of nonsense, airing inane profundities, in the manner of certain fashionable French postmodernists, on the contrary the author is very sincere in trying to explicate the issue of Platonism in Mathematics, but what gives to his effort an almost comical touch, is his very sincerity, the idea of clarifying such matters with logical chains of pure arguments. The impression his sustained effort produces in the present reader is one of pedantic pedestrianism.

Why is the notion of Platonism so abhorrent to most people? In fact it is not unusual for people to express strong Platonist views, yet with the caveat of not really being Platonists themselves. Could it be that Popper, who otherwise temperamentally fits so well into the Platonic tradition, disparaged the ethics of Platos political ideas as crypto-fascist? Or that Platonism in one of its most vulgar manifestations (and of course what goes for Platonism varies greatly, different shadows cast by Ur-Platonism) appears just silly in its insistence of rigidifying ideal objects.

The essence of Platos theory is the existence of abstract entities. Entities that are manifested in our spatio-temporal world, but somehow transcend it. As a metaphor, the Platonic picture is indeed very suggestive (as well as self-referential), but it has provided a very successful paradigm for basic natural science, in which explanations are never given on the level of concrete manifestations, but given their power by being conceived abstractly, revealing hidden mechanisms. Platos philosophy goes back to the older Greek tradition exemplified by Parmenides, that taught us to distrust the multifarious and confusing world of the senses for a simple and unifying world of principles. To make this concrete, one may refer to a query that I guess most inquisitive people must have encountered in their childhood. Is it true that 'red' appears 'red' even to my neighbour, or does 'red' appear 'blue' to him and vice versa? The next step is to compare the view of the world of myself with those of other minds? Are they the same? Does this question even make sense? Leaving the notion of Solipsism aside (a seductive, if not always attractive alternative for anyone enamoured by the principle of Occams razor) one clearly realizes that 'sameness' can only be understood in the sense of 'isomorphism'¹. It does not really matter how things occur to us, as long as one experience of the world can be decoded in the other. This induces a notion of equivalence between different sensations, making communication and sympathetic understanding feasible. Thus ironically one is soon led to the conclusion that it is really only abstract entities that can be directly compared when it comes to different minds. Whether red is 'red' or 'blue' is an unanswerable question, but the equality of two colors is something entirely different and would not differ (at least not in any sense which

¹ This notion of the subjective experience of the world, and its uniqueness, is always very much on the mind of Frege

is easy to imagine²) between individuals. Thus only in our perceptions of abstract entities is there a direct unity. This I believe is one of the central themes in Platonism, and one insufficiently appreciated and hence the source of so many vulgar misapprehensions.

One major misapprehension in trying to understand Platonism is to 'objectify' those abstract entities and ask to what extent they really exist. If they exist they clearly cannot be ordinary objects caught in time and space, but some ghostlike objects residing in another realm. And so the occasion for the classical argument against 'Platonism' as expressed by Benaceraff, namely how can we finite human beings possibly attain knowledge of those arcane objects? What kind of communication lines afford such contacts? If mathematical objects exist outside time and space, and are inert to boot, clearly they must remain inaccessible. But to me this amounts to a naive 'objectification' a primitive inability to really appreciate the nature of abstraction. Abstractions are extracted from concrete tempo-spatial entities, this is in fact how we come to apprehend them, and this means that although they cannot be pinned down in any spatial and temporal way, they nevertheless exist (in fact as we have argued above far more directly in a sense than mere concrete objects) and far from being disjointed from the concrete sensual world, Plato argues that they form it³. Hence the objection of a Benaceraff appears naive, not to say paradoxical, as if made in black jest.

The author describes two versions of Platonism and anti-Platonism which he claims are the only tenable ones impervious to all the classical objections. He does not come down one way or the other, refusing not only to make a stand, but claiming that it is in fact impossible to make a choice, and that this has significant meta-physical consequences, which he takes quite a delight in expounding at the end of his book.

Now it is not hard to understand why the author does not commit himself to a choice, in fact the two approaches are for all intents and purposes completely identical, with exactly the same consequences for the practice of mathematics. According to Balaguer FBP⁴ means that any logical consistent axiomatic theory is confined by the existence of abstract mathematical objects that fit its rules. His version of anti-Platonism, on the other hand means that any such consistent theory is a fiction, in the sense of referring not to anything existing, but only being, like a novel, internally consistent. Thus anything goes, as long as it does not lead to contradictions. The difference being that in the Platonist version, existence is granted somewhere in some mathematical realm, while in the anti-Platonist version, this realm is purely imaginary. As the realms are equally inaccessible, in practice, as noted above there is no actual difference between the two⁵.

To a mathematician Balaguers version of Platonism (his FBP) sounds suspiciously

² I am of course leaving aside the obvious objection presented by the phenomenon of color-blindness, but it should be clear to the reader what I am driving at

³ This incidentally dovetails very well with contemporary physical explanations of the universe.

⁴ The author has a fondness for acronyms of which his text abounds. FBP stands for Full Blooded Platonism

⁵ The author adheres to what he calls a gentler 'Logical Positivism', the original notion having been discarded in philosophical circles, and hence no modern philosopher wants to have any truck with it. To the casual reader it is not so easy to understand how the authors version differs from the classical, and how it should be more congenial to metaphysics.

like formalism as proposed by Hilbert. Mathematics is but a game played by arbitrary, but consistent rules, and the main point is that the 'internal' properties (whatever that means) of the objects play no part, only how one operates with them (structuralism). To a mathematician there are two problems with this. First by doing away with tangible mathematical objects, they nevertheless reappear as formal systems (but as such far less congenial to the mathematical mind). And secondly, as Gdel showed, the question of consistency is more or less intractable, thus making the authors claims essentially vacuous.

As Balaguer admits, his notion of mathematics, is essentially post-modernistic. Anything goes, it is just a matter of the particular axiomatic conventions. No theory is intrinsically less true than another, what makes the difference is ultimately a question of taste.

Mathematicians view of mathematics is different. It is certainly true that in practice matters such as importance and beauty play almost as crucial a role as mere correctness. Mathematics is a human endeavour, subject to fashions and the formative influence of history. It does not proceed randomly, but grows organically in response to natural questions. Of this there is little controversy. But mathematics differs from art and language in being stubbornly independant (as anyone struggling with inequalties that keep going the wrong way can testify) to our wishes. To the working mathematician mathematics has indeed an objective existence ultimately independant of the human mind. This is a conviction that grows on him, in a very similar way that the existence of a real world out there grows on us all through our incessant interactions with it. The distinction between the two worlds is in fact hard to make once you are a committed Platonist, as both are ultimately based on transcendant abstract principles and thus liable to be fused.

There are of course phenomenon which forces the mathematician to make some kind of distinction. The handling of infinite entities is something that appears to have no tangible physical counterpart, and provides as such a mathematical object. But as noted above, talking about objects when abstract entities are concerned is somewhat of a contradiction in terms. We all seem to have a very strong intuitive notion of the natural integers, which is an abstraction we are able to form very early. Thus it is very dissatisfying to uis to be told that the integers are but equivalence classes of finite cardinalities, or that they are, say in Neumanns terminology but the sequence $\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\} \dots$ clever as it may be. Clearly there are so many different 'object'-realizations of the integers, and clearly none is the 'right' one⁶. In fact it is in the nature of abstract objects that they cannot be pinned down and 'objectified'. Thus the belief in the natural integers is a deeply intuitive matter, and as such the notion of the integers appears as a primitive notion in the sense of not being reducible. Also the Peano Axioms post-date, not only historically but logically, the notion of integers, and are seen by us as being interesting only so far as their structural properties complies with those we would find appropriate to the numbers⁷. Essential Platonism is about Truth (was there ever a more abstract idea?) and Meaning, none of which really emerges in the so called full-bloodied version championed by the author. And clearly

⁶ It belongs to one of the major misunderstandings of Platonism, that every kind of notion is represented by a canonical form.

⁷ In one of the few interesting asides Balaguer points out that the integers have very many properties that are not listed among the Peano Axioms, like being 'non-red' e.g.

one cannot argue about such metaphysical matters in a linear logical way, circularity is inevitable (as is infinite regress).

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