**The Best Writing on Mathematics in 2009**

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The World of Mathematics was a famous and very successful anthology in four volumes on Mathematics edited by J.Newman in the 50's. I believe it is now out of print since a long time. It was translated into many languages, and had quite an impact, especially among non-mathematicians of wide interests. In fact the purpose of the anthology was to present mathematics as an inescapable part of human culture, especially Western. Thus the articles were not technical but addressing a wide variety of writing with connections to mathematics. They were arranged according to different themes, and Newman wrote extendend commentaries on them. I in particular remember with cherish a chapter ' 'On Magnitude' of the classical book by Darcy Thompson with the title 'On Growth and Form'. Some simple mathematical principles of scaling were shown to go a long way.

The present anthology is meant to be an ongoing affair, one volume to appear for every year. The initiative is commendable although the task is not that easy. The problem is that the sense of there being a mathematical culture has declined in recent decades. One may speculate as to why. I suspect that the true interest in popularization of science has declined. Of course books addressing the layman are still being published, but the motivation is more commercial than educational, reflecting a general decline of the publishing industry. Another reason may be the increased level of formal education among the population. People no longer feel the same need to educate themselves, of having unsatisfied curiosities satisfied, of wanting to compensate on what they may have missed out on. Now if people read popular science it is more out of a desire to be entertained. Scientists may also be culturally less sophisticated than they were in the past. In the 19th century it was expected that you could play an instrument, be a passable draughtsman and also write some poetry. In short there was a time of less specialization. Nowadays a scientist is under great pressure to produce and publish, there is little incentive, material or otherwise, to engage in cultural reflection. All said and done it is much harder to find suitable material, especially if you confine yourself to a given year, which Newman did not do at all.

Now any anthology is bound to disappoint as well as to delight. Personal taste makes a big difference. I must say that, with one notable exception, the articles collected under the theme mathematical education I found hard to read. To be honest I tend to find them rather scientifically pretentious, too often showing off the garb of science without the motivating content. Likewise I was less than enchanted by some of the philosophical articles as well. The reason is that education is an art, and philosophy is a form of poetry. The poetry of science, as I usually refer to it as. Philosophy is more enlightening when it proceeds less by argument than by evocation. The reason is that any philosophy worth its salt dwells on the border of meta-physics. Thus it cannot be studied systematically, trying to catch the flights of philosophical fancy and tame it, is like weighing the mist of dancing fairies. It simply misses the point. Thus the long article on the shortcomings of a textbook explaining what is a tangent is a case in point [1]. Nothing in it is factually wrong, still it seems to overlook an important point. How do we learn things? This is a subtle process. It would be naive to assume that it is based entirely on a bottom-up approach. The idea of a tangent is very intuitive, and anyone with a mathematical bent will recognize it as soon as it is encountered. (Provided of course that he or she is ready for it). The fact that the definition may be wrong and not allow for the case of the tangent intersecting the curve at some other point but claiming that the tangent is on one side of the curve only, is irrelevant. If you get the point you realize that the notion is local. To make that precise you would need a slightly more involved notion, which would only make it even harder for someone to learn the concept by painstakingly making sense of the definitions. Mathematicians make many mistakes such as those when talking about things. To point them out would be considered pedantic. There may be a paradox here, after all mathematics is supposed to be the most precise of sciences; yet to take things to literally is always a sign of autism. When it comes to mathematics I fear that a large fraction of the population is autistic, in the sense that they cannot read between the lines, but takes everything literally. In social situations this is rare, but in mathematical common. Even the mathematician may be in the same situation, when trying to make sense of something for which he or she is not yet ready, having no a priori understanding to bring to the scene. But maybe the worst thing is that the authors reveal a rather striking mathematical naivety when they point out that a tangent touching a curve has (at least) one point in common with it, but a finger touching the page of a book, does not have a single atom in common with the page. Of course with some basic underlying sophistication, this example might be a useful illustration of the difference between mathematics and the real world, pointing out the nature of a mathematical point. In the way the authors point it out, one almost suspects that they think of mathematical points as tiny particles!

Another case of this kind of autism is the article on Aesthetics in mathematics[2]. It is a lengthy one, and in my opinion it really does not go anywhere. Of course that does not mean that I do not agree with some of the remarks, the fact that there are no real mathematical critics as there are literary critics, I have often lamented. But the perspective of the authors is that of evaluating papers on aesthetic ground, while mine is on evaluation on mathematical importance, although admittedly the two aspects are very much related, but not always coincident. Sometimes an ugly proof could be more fruitful and instructive than an elegant one. Now this paper also seems to miss the point in trying to pinpoint what is the aesthetic element in mathematics through making it too concrete. Nice two-dimensional patterns may of course reflect mathematical beauty, but most of it cannot be manifested in this way. When Hardy speaks about patterns, he speaks in a very abstract way. Mathematical beauty verges on the meta-physical and any attempt to quantify and demystify it only makes it slip away from the net. One gets the idea that the authors have never really had a very personal and intimate relation to mathematical beauty, and they look upon it with a certain suspicion, almost verging on hostility, when they relate it to elitism. They see themselves as hard-nosed scientists, trying to make precise a phenomenon, and thereby stripping it of its pretensions. Attempts to reduce the

[1] Mathematics Textbooks and Their Potential Role in Supporting Misconceptions

[2] Aestethics as a Liberating Force in Mathematics Education

beauty of visual arts to some quantifiable parameters[3] or music to repetitive patterns of symmetries, something which the music of Bach naturally lends itself to, are more or less useless. The appreciation of mathematical beauty is some kind of epiphenomenon, which like faint starts is not visible in the center of your vision, only imaginable at the periphery of the same. One could as well speak about the happiness that mathematics brings in some systematic way. While truth may be an intrinsic property of mathematics, beyond our control, mathematical beauty may very well only be present in the eye of the beholder. Thus it is not really part of the Platonic essence of mathematics, although of course it poetically very much enhances that Platonic essence.

In an article on analytic versus intuitive thinking[4] the claim is made that our intuition about probability is defective, and some evolutionary explanation is given. I am very suspicious of the 'so so stories' concocted by evolutionary psychologists, doing seriously what Kipling did tongue in cheek. To discuss the notion of probability one needs to make things a bit more precise. A statement such as the probability of X is y is usually meaningless. What is the probability of a man becoming a hundred years old. In order to make sense of it you have to make more specifications, and usually if those specifications are made, the problem dissolves itself. The question of probability is similar to the question of average value. What is meant by an average value? The naive may think that this is something canonical, but far from it. There are many different kinds of average values, all depending on the context. How can you speak about the intuition of average values when they can be so many things? To make the point more specifically let us discuss the issue brought up by the authors.

*If a test to detect a disease whose prevalence is 1:1000 has a false positive rate of 5%, what is the risk of a person tasted positive to actually have the disease?*

Many people confronted with the question would say 95%. This is wrong, but why is it wrong? What does it mean to say that we have a false positive rate of 5%. Does it mean that of all the people testing positive 5% are testing falsely, i.e. do not have the disease? If so it means that 95% of people testing positive are actually stricken with the disease. Or does it mean that if a test is made on unafflicted people, there will be 5% testing positive. If so of a thousand people you expect 50 to be tested positive, and thus out of those only one affected i.e. one in fifty that is 2%[5]. Honestly it is not clear what is meant. It is just as with averages, there is no definite meaning to what such a statement refers. Once you make the meaning precise the right answers follows whether or not you know anything formally about Bayesian reasoning or not. It is not a question of analytic versus intuition it is a matter of common sense spelling out what the terms means. If you do not know what is meant the question has no answer. Of course this does not mean that there are many cases where your expected and intuitive reasoning in probability would be wrong, it simply means that this particular example is not well-chosen.

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[3] I am reminded of Klein trying to make sense of the beauty of a classical Greek sculpture by employing differential geometry and tracing lines of curvature.

[4] Intuitive versus Analytic Thinking: Four Perspectives

[5] The reasoning is a bit sloppy, out of 1001 people you expect 50 to be falsely pointed out and hence of 51 positives only 1 actually sick which gives a slightly lower chance. A mathematician making a rough argument would not be pedantic, especially as the figure 5 is bound to be approximative anyway

Although I am critical of much so called research in mathematical education does not mean that I think that there is no place for it in an anthology on writing on mathematics. On the contrary, even for opponents of mathematical education, or especially for the skeptics, it is of great value to be presented with representative samples. The literature is so wide, and so much of it is substandard (admitted by education people themselves) so an editor really does a great service selecting the highlights. And of course reading articles with which you do not agree is usually more instructive than nodding over things from which no one would think to differ.

The same kind of criticism levied against the Educational articles can also be levied against some of the philosophical. It is well-known in the history of mathematics that much of mathematics, especially early calculus was not done rigorously yet yielded true results. As to make a formal address of this [6] and discuss whether or not inconsistent objects can be thought of existing is also to miss the point. Clearly there was something going on, and the various formalizations did not really capture it fully. They were not only incomplete but also defective. This is something which is very interesting and intriguing, and shows how the intuition of inspired people may go beyond formal inadequacies. To make this formal by speaking about some kind of formal consistency of the inconsistent is a case of busy-work, a disease brought out by the pressures of identifying a new discipline with scientific credentials, in this case mathematical philosophy. Once the meta-physical aspects of philosophy are rendered mundane, the poetry is squeezed out, and you may very well ask what is left. Some people may claim a good deal, I am not so sure. The general attitude of mathematicians towards philosophy is that it should be done informally over coffee[7]. This is not as disparaging as it may at first seem, it only means that you should not see the pursuit of philosophy in mathematics as yet another mathematical discipline. It is a reflective and evocative pursuit, and one in which you engage because you cannot help yourself, not because you want to write a publication and go to a conference. In philosophy your object is to find new ideas, new perspectives, and also to find striking new formulations. Those ideas are not to be tested as ideas in science are, because in science there are over-riding paradigms into which things need to be fitted. Philosophy is in essence meta-physical, this means that it should try and transcend rigid paradigms.

Now I mentioned one exception, and that is the article on curvature[8]. I do not understand why it is grouped with the papers on Mathematical Education. It is a very nice paper on some particularly sweet mathematics which is also widely accessible. True it has great educational potential showing how rather deep ideas can be made manifest in simple practical ways. If all papers on Mathematical Education would be like that, the field would be a gold mine. It is exactly the kind of thing that would have fitted into a Gardner column.

In fact there are many articles touching nicely on philosophical issues. I may not always agree with the opinions, e.g. I do disagree with Devlin when he claims[9] that the

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[6] Applying inconsistent mathematics

[7] Suggested by Raghunathan in an interview with him, and indeed being very representative of the attitude of mathematicians, expressing not so much disdain as humility.

[8] Exploring Curvature with Paper Models

[9] What is Experimental Mathematics

added emphasis on calculation makes mathematics less Platonic, I would argue the other way. It highlights how mathematics can be separated from human thinking on it. But such coffee-table discussions of the philosophy of mathematics do not lose sight of the point of the exercise. They may provide nothing but opinions, on the other hand how could you prove things in mathematical philosophy, if you could, you would according to Russell make it part of mathematics[10] Thus I very much appreciate the contributions by Chandler and Nathansson[11] written in that spirit.

There are two many articles to be reviewed individually, so let me just remark that I found the exposition [12] of alternative ways of how to represent real numbers by computers very instructive, using the notion of repeated exponentation. A real gem. The rambling reflections of a Philip Davis[13] I found charming, and maybe the best written piece of them all is by Freeman Dyson[14]. It may not be instructive in any specific way, yet it is delightful nevertheless, conveying a sense of mathematical community. The kind of meta-writing on mathematics and its practice we cannot have enough of.

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[10] Actually Russell saw philosophy as the mother of all the sciences. As soon as a part of philosophy became precise enough to allow arguments it became part of science. Thus as history evolved, science became bigger and bigger, while philosophy was draining like a pond.

[11] The Role of the Untrue in Mathematics, Desperately Seeking Mathematical Proofs

[12] The Higher Arithmetic: How to count to a Zillion without Falling off the Number Line

[13] Bridging the two Cultures: Paul Valery

[14] Birds and Frogs