Philosophical Foundations of Physics

An Introduction to the Philosophy of Science

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`October 1 - 3, 2008`

Carnap was very active in the Vienna Circle of the 20's [1] a group of like-minded philosophers intent upon doing away with the last vestiges of metaphysical garbage in philosophy and to set philosophy on a firm scientific basis, which meant above all to create a precise and formal language in which meta-physical nonsense would be much harder to perpetrate and in which many silly disputes in philosophy ultimately based on ambiguity of language would find their resolution[2]. However, laudable as such ambitions were, in retrospect the Vienna Circle has gotten somewhat of a bad reputation as a bunch of positivists, looking at the world too blinkered, and as Popper has remarked, their program of anti-meta-physics was ironically based on a meta-physical conviction[3].

The present book, and the only one providing a non-technical account of Carnap's philosophy, was conceived on the initiative of Martin Gardner, who in the forties had been subjected to Carnap's lecture and when those were repeated during Carnap's exile to Los Angeles was instrumental in having them taped and subsequently transcribed taking a very active part in the editing of an oral documentation into a written form. One may be excused for suspecting that much of the readability of the text and its lack of technical philosophical jargon is due to the intervention of Gardner. In many ways the text reads as if it is addressed to children, and when philosophy is concerned that is not such a bad idea, even if a more sophisticated audience is kept in mind. Unlike science, which is accumulative, philosophy thrives on the fundamental inescapable questions, which have to be reformulated anew for each generation.

Carnap starts out by stating the basic aim of science, namely the formulation of universal laws. A universal law he reminds the reader is of form $(x)(Px \supset Qx)$ which in English roughly translates into 'if P holds then so does Q. To a modern reader this seems a quaint bit of pedantry, as if the formal expression somehow would carry more exact and instructive information than its intuitive illustration, but that is a very minor quibble. The basic questions in the philosophy of science are how one finds and formulates such universal laws and how one proves them. Carnap does not address this question but rather slurs over it. As to the first he makes some reference to observing regularities in nature of specific

[1] Known as the Wiener Kreis. The Circle as a social gathering came to a tragic and premature end through the shooting of its leader Schlick by a demented student

[2] This ambition did of course pre-date the circle, similar ambitions were voiced by C.S.Peirce in the late 19th century, although the latter due to a complicated personal life was not able to pursue it systematically

[3] As I do not tire of reminding readers, Collingwood (as well as others before him) have remarked that the rejection of meta-physics is by itself a meta-physical stand.

facts[4] and then making generalisations, i.e. arguing from the specific to the general known as the inductive method, rather than arguing from the general to the specific. The process of induction is of course inevitable in any scientific pursuit, yet the general description is somewhat misleading giving the impression that science is about looking at data and then discovering regularities. Now admittedly Carnap refines the notion by referring to experiment, the precise process of putting well-formed questions to nature and interpreting the answers[5] . However, he does seem to think of experiment in too narrow a sense, namely that involving controlled experiments, which in a quantitative setting means controlling all the variables except a few. In this way he disqualifies Astronomy as an experimental science as well as pointing out the insurmountable difficulties, not the least moral and practical, of performing experiments in the social sciences. The latter is of course a very valid point, but the impossibility of controlling the situation does not mean that valid questions cannot be asked. After all, as we will return to later, the confirmation of Einstein's theory by exhibiting the expected angular shift in the position of a star, is a good example of an experiment in astronomy. True we cannot make an solar eclipse by will, but we can take advantage of it. Similarly in the social sciences we can exploit previous situations[6]. Still the true nature of science is not the establishment of so called empirical laws, as will become apparent later in the book.

However, when it comes to the proof of inductive laws we encounter a very serious problem first clearly formulated by Hume[7]. Any law has an infinite number of consequences and an inductive verification would in principle involve an infinite number of confirmations which obviously is not feasible. It is not that Hume, except when he had his philosophers hat on his head, doubted many inductive laws, the acceptance of which is necessary to go through the daily business of our lives (and thus done tacitly without us being aware of the fact); only that he pointed out that the principle of induction could not be proved, it was in a sense a meta-physical assumption[8]. This problem is indeed very serious and inescapable, and the attitude of Popper is that we can never confirm empirical laws (or any non-trivial laws for that matter) we can only contradict them. Thus Popper's meta-physical attitude

[4] Carnap restricts the notion of a fact to single unrepeatable occurrences that can be noted, observed and documented, and he is right in doing so. Ultimately science is bound to respect facts, those are the basic ingredients of the empirical reality to which theories ultimately have to conform.

[5] The explicit formulation of this principle is usually attributed to Francis Bacon and as such meant to make a distinct break from previous purely speculative endeavours as practised by the Greeks. However, Aristotle also performed experiments in contradistinction from Plato who took a more exalted view. And the classical story of Galileo refuting Aristotle by dropping two unequal weights from the leaning tower of Pisa appears to be apocryphical, instead what Galileo had in mind was some kind of thought-experiment. What would happen if two equal weights were connected by some thread?

[6] It is of course morally reprehensible to affect a famine, yet when a famine has occurred through no fault of our own, we can search the documentation of it for possible answers to questions we may pose.

[7] Although Hume is referred to appreciatively in context of cause and effect, he is not invoked in this matter which is intimately connected with the former.

[8] Not that Hume approved of meta-physics, on the contrary he ends his famous book by noting that any book solely concerned with meta-physical speculation deserves to be burned. (It is not entirely clear whether he exempts his own book.)

to science is that it is provisional, the findings of tomorrow may contradict established orthodoxy of today, but nevertheless our pursuit involves an asymptotic approach to truth. Now Popper's remarks are hardly original, they were of course formulated also by Hume, and Carnap also refers to (somewhat confusedly in my mind) a difference between certitude and truth, apparently assuming that basic laws could be true if not certain! (This pragmatic and instrumental attitude to Truth he clearly shares with a previous generation of anti-meta-physical philosophers, especially the Americans, C.S.Peirce, William James and John Dewey, and as such it is probably more or less inevitable to anyone committed to an anti-meta-physical stand.) But the point is that Popper makes this generally accepted view a cornerstone in his philosophy of science, from which a hoist of other consequences follow, in particular the focusing on falsification (a law can always be contradicted by a single obstinate fact, although of course in practice matters are seldom so clear-cut, but Popper is not out to provide a procedure for doing science). Now Carnap is also fully aware of the criterion of falsifiability when he speaks later of analytic truths as being non-contestable, meaning that they have no possibility of being false, and hence tell us nothing about the world, a matter to which I would like to return later.

Now unlike in deductive mathematics[9] we can never speak about certainty in the real world, thus the notion of probability is crucial. Now the nature of probability is at least on first sight very intuitive, which means that it is hard to give a formal and yet encompassing definition of probability without getting embroiled in circular reasoning. In particular what does it mean to say that a certain event has probability 0.7 say. How should we interpret it and how should we let it influence our decision making? One may claim that it is always a good thing to choose the most probable event in the long run, but what does that mean? And often we do not have the option of a long run, for individuals the choices we encounter are one-time only. And how do you compute such a probability? How do you compute the probability of there being extra-terrestrial intelligence? Clearly a probability estimate is always contingent upon available information. What is the probability that I will die within six months? If the only available information is that I am physically present in a hospice, the probability is quite high, but if I would remove myself to a day-care centre, the probability would dip drastically. So hence there is a difference of asking the question in the abstract or in a contingent way[10] and even in the contingent way the calculation of a probability is very problematic. Now there is in modern probability a formal solution to the problem. In effect it is reduced to a question of measure theory with pre-determined probability distributions. This formal machinery, completely adequate (and necessary?) for a mathematical treatment (mathematical probability) is in the nature of analytic truths, and the challenge for a synthetic probability theory (statistics) is to relate the simple elegant mathematics to so called real life. In practice this philosophical dilemma is solved by considering the mathematical set up (with given distributions) as mathematical models, testing their confirmation with empirical data, and then modifying them correspondingly (so in this sense we can speak about the probability of a given probability distribution). To do those manipulations appropriately requires on one hand an analytic skill in formal

[9] If even there!

[10] If we assume a deterministic world as indicated by classical physics, clearly when having complete information the probability is either one or zero, but which one?

probability as well as common-sense, and the latter is as essential in any business with the real world as it is resistant to formalisation. The bottom line in all research of a statistical nature is the problem of assessing probability numbers. If a study shows that a certain vaccination is say 97% effective, should we then implement it, because of that high certainty (which of course in itself is approximative and may be all wrong with a risk of say 3% and so on)? What confidence intervals should we use? And is that not just a convention? Of course taking the convention too literally leads to nonsense. Every suitable long sequence of events in a personal life are statistically very unlikely, should we hence doubt their occurrence? In countless trials with extra-sensory-perception gurus, there are bound to be sequences of coincidences which individually cannot be explained by chance (in the sense of probability being too low), yet there is a fundamental difference between saying that a lottery has a winner and that a specific individual will be a winner. If you win on the lottery, or somebody you know, you consider the event very unlikely, yet you are not surprised if somebody wins the lottery. The difference being that in the first case we do specify the individual, in the second we do not, we only know the winner after the event not before. This ties in with concepts like entropy which lies on the basis on any statistical conformity of laws, while the numerical conformity is the bane of any non-conceptual science, such as that of clinical medicine.

On those matters Carnap does not have very much to say. He does refer to Mises frequency definition of probability as a link to the empirical world, thus the probability of a coin being tossed head can only be determined asymptotically by a potentially infinite number of tosses. This is of course a common-sense approach useful in practice, but of course theoretically it is unsatisfactory. After each toss the coin will change a little, and after a few billion tosses it might have worn down to nothing, not to mention the state of the physical universe in the end, would the coin be more durable.[11] However Carnap makes a distinction between two kinds of probability, referred to as inductive and statistical probability, roughly corresponding I believe, to what I have just sketched[12].

Carnap wrote his doctoral thesis on the nature of space, and what he writes here is perhaps the most interesting part of his book. In discussing those themes he embroils himself in the kind of technical depth that you would not expect to find in the writings of Popper[13]. One basic component of fundamental science is quantification, an activity

[11] What is the probability that the sun will no longer set and rise from a given position on Earth? We 'know' from Newtonian mechanics that eventually the rotation of the earth will be synchronised by its orbital rotation, as is the case of the Moon around the Earth. Thus like for the case of the infamous turkey of Russell there will eventually be a day of recogning. The inductive evidence for a return of the sun will be overwhelming (I guess we are talking about a few trillion occurrences) yet we believe the Newtonian prediction. On what inductive evidence? Not any direct, and even the indirect, such as the repeated confirmation of Newtonian mechanics in practical life, will not amount to the same. Clearly there are other principles of conviction involved here, to which we will have occasion to return.

[12] It is not at all clear from the mere notation how to make the correspondence, but I would suspect that statistical probability refers to formal mathematical, and inductive to what I call statistical and empirical, but I may have missed some point

[13] It could well be that Carnap was a more accomplished mathematician and physicist than Popper,

often resented[14], and quantification in science and its measurements are ultimately reduced to the measuring of lengths, and hence an inescapable character of the space we inhibit. Carnap sees three levels, the purely qualitative and descriptive (that stone is hot), the comparative (that stone is hotter than the other stone) and finally the quantitative (the temperature, what ever we mean by that, is given of a stone at a certain time). Now the opposition between quantitative and qualitative maybe involve a confusion of categories, it is not simply that quantitative is a refinement of the qualitative, qualitative reasoning in science is fundamental, the very act of quantification is a qualitative process.

Now measure is ultimately connected with the method of measurement, and if you would be pedantic, as some people apparently are (if not Carnap) the concept of what is measured differ depending on the methodology. Thus in particular we do have different notions of what corresponds lengths, one notion when we measure small distances with a ruler at home, quite another one when we measure astronomical distances. The notion to which those different distances are related is a fascinating one, and will be returned to later.

First take the measurement of time. This is a crucial one, and even if we have great difficulties as did Saint Augustinus, to make clear in our mind what time is, we can always take an instrumental approach by specifying a method of measuring it and thus disregard the meta-physical complications. The obvious problem of time measurement is that time has an arrow, we can only flow forward never step backwards, thus we can unlike lengths, never directly (and this is also a problem as we will see) compare different time intervals, nor can we add two time-intervals by lining up one endpoint with the starting point of the other, except in particular circumstances. One such particular example is the phenomenon of periodic occurrences, which ideally continue indefinitely. We then say by fiat that the interval between any two occurrences is of equal time-length we have established a method of measuring times in number of beats. (Thus in a sense only a discrete unit of time). One may naively object that this definition is stupid, one should never involve erratic periodic behaviour (such as the emergence of a certain Mr Smith out of his house) only very regular (and quick) periodic behaviour. Of course how can we measure the time between pulses, if we yet do not have a method, we simply need to find a starting point. Now from a purely formal point of view (consonant with fashionable post-modernist drivel) any periodic phenomenon is as legitimate than any other. Carnap does not in principle deny this, and takes an amusing example what would happen if the time unit was based on the pulse of Mr Smith? Whenever Mr Smith was running or running a fever, all other processes

but of course this is ultimately beside the point. Rational reasoning, William James reminds us, is based on picking up the relevant facts out of a phletoria of present ones: The same thing goes for philosophy. Many accomplished scientists are poor philosophers.

[14] Carnap just points out that quantification is just something we impose on nature, it is not just one aspect of nature we single out, ignoring the rest. Goethe famously objected to the quantification of nature, especially in his 'Farbenlehre' in which he opposed Newton. According to Goethe nature should be studied under natural conditions, not twisted to our purposes. We should take it in sensorially not by quantitative thought. Here Goethe shows himself an ardent anti-Platonist more concerned with the delightful surface phenomenon of nature as it is known to us through our senses than in any deeper hidden sense. Not surprisingly Goethe was far more successful as an anatomist than as a basic scientist

in the universe, such as the rotations of galaxies would slow down. This would be very gratifying to Mr Smith, but what would happen when Mr Smith died, would time stop? (From the ego-centric perspective of Mr Smith this would be true and he would literally not care any longer.). The imposition of a quantification of time is clearly a convention and it does in no way change the universe, only our conception of it. Now is there some kind of Platonic notion of time? Conventionally not, yet in science we choose systems of measurements which makes our conception of it as elegant and useful as possible. How to do this is clearly a very non-trivial problem, yet it is one that is addressed by any serious pursuit, indicating that there are after all some meta-physical motivations in science. As it turns out there are a lot of regular periodic phenomena which corroborate each other providing useful and natural definitions for measurement, and hence the mathematical correspondence of time with real numbers [15]. This correspondence is never ultimate, as science progresses more and more fundamental correspondences can be formulated, but this is not a problem, just as the impossibility of exact measurement should be a reason for no measurement at all, rather than providing an incentive for the kind of asymptotic approximation to the ultimate one, to which we have already referred in a more abstract context.

Ultimately the measurement of time (or temperature, or current etc) are reduced to the measurement of length (in the case of time we refer to a clock), thus it is natural to concentrate on the nature of space.

There is a fundamental, but by educated people not always appreciated difference between counting and the measuring of length[16]. The measuring of length first involves a prototype, namely the selection of a unit length [17], then it involves the principle that a unit length can be moved around without changing its length. In practice in the physical world how do we know that a rod is rigid? This is similar to the case of periodic behaviour, we could always assume that it is and go from there. However, it turns out that some choice of so called rigid rods makes for a more convenient notion of lengths and the laws of physics. In particular rods made of metal show (as it turns out under constant temperature[18]) a high degree of mutual correspondence[19]. Now from a strictly mathematical point of view what does it mean to change position without changing distance? We simply propose a

[15] Of course in practice all the numbers we involve us with are counting numbers and hence rational by extension involving small denominators

[16] Mathematically educated people often see the integers as embedded in the real line

[17] strictly speaking we do not have to assign the value one to such a prototype, we can assign any number to it, in fact in Euclidean geometry there is no natural choice of unit for length, but there is a natural unit for angular extension. The full circle was assigned 360 by the Babylonians, while in modern mathematics we find it more convenient to associate the incidentally transcendental number 2π

[18] How do we know that metal rods change their lengths under variation of temperature, when the measuring rods also do so. The solution is due to the differential change of thermal extension for different substances, the mercury thermometer works because the thermal extension of the glass container does not tally with that of the mercury pillar inside.

[19] This can of course be compared to the notion of currencies. Those are not rigid, varying not only in space but also over time. How do we make sense of a constant notion of monetary value, similar to the notion of an objective length? Formally we could just take one particular currency, but that would show

group of rigid motions, involving the compositions of translations and rotations, which of course can be explicitly represented by matrices, once geometry is artithmetized as tuplets of real numbers[20]. Now with the notion of a rigid ruler [21] we can measure small distances by simply reading off or extrapolating by successive translations if necessary. But what about large distances where it is impractical to use the naive ruler methods? Then we use triangulations based on the theorems of Euclidean geometry, the details of which are clear to any educated person. But now we are using a different methodology how do we know that the two notions of the concept of lengths do agree? Now this suddenly becomes an empirical question, geometry is no longer just a mathematical abstraction, it relates to the real physical world, and it becomes an empirical question. It could mean that the two notions of lengths do not coincide, then what do we do?

As remarked by Poincaré, we could simply keep Euclidean geometry and then give up the idea that the rods we have been using for measuring lengths are rigid after all. Or if we feel more comfortable with the physical rigidity we will have to reconsider Euclidean geometry as a accurate description of reality. Now this attitude of Poincaré, to some extent presented tongue-in-cheek, referred to as conventional may strike many as shocking. It seems to relativize truth, in fact it seems to make the process of falsification impossible, whatever we like we can keep. However, this is of course not the case, there are still constraints to which we have to adhere, we cannot both maintain the rigidity of a measuring rods and the Euclidean nature of our space. The point of Poincaré is to show that a description of nature can be made in several isomorphic ways, in either description there will be empirical checks, but those will be differently if isomorphically formulated. Thus Poincaré simply points at the inevitable impossibility of distinguishing between isomorphic theories, familiar to all mathematicians. As the notions of our theoretical descriptions are instrumental, i.e. dependant upon ways of measuring, there really is no alternate way of anchoring physical theories [22]. Thus what we are doing is not relativizing truth, only lifting it up to a higher conceptual plane, in the true Platonic spirit. Our choice is to some extent a matter of convenience, depending on what makes most sense and allows a most elegant explanation[23]. Such ideas did not of course originate

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similar idiosyncratic behaviour as that of Mr.Smiths pulse. True some currencies are thought 'harder' than others, and there may even at least for some time be some correlation between different hard currencies; but such will not sustain themselves over time. Now can one really make sense of an absolute notion? Economists differed. One way of solving it was the Gold standard, decried by less sentimental and hard-nosed economists such as Keynes. The common consensus nowadays is that such a notion does not make sense. Money is indeed a social construct and cannot be fixed outside the hopes and desires of traders

[20] As a fact of curiosity, in classical geometry, only constructions using rulers and compass were allowed, in such a geometry it is enough to consider the universal quadratic extensions of the rationals, i.e. the union of all repeated quadratic extensions. The aritmetizations of non-Euclidean geometries are slightly more subtle, but not much, as we do not have Cartesian co-ordinates, but polar ones still make sense.

[21] having it suitably marked with 'equidistant' notches does not hurt matters

[22] One may compare to the case of comparing different sensory impressions between different individuals, there is no way such can be directly compared, all we can say that to a very high degree those are isomorphic between different individuals otherwise intra-personal communication would be impossible

[23] Do objects get smaller as they move away from us? Unless we happen to be an ego-centric child,

with Poincaré, but due his superior mathematical and physical sophistication he was able to present the case more interestingly and provokingly than his predecessors. To take an earlier example is the old controversy between whether the Earth rotates around the sun or vice versa. From a purely mathematical perspective, either is equally valid, rotation around something being a relative notion. And in fact the Catholic authorities had no quarrel with a Heliocentric world-view as long as it was considered a mere mathematical conventions useful for calculations. However, to really assume that the Sun was the centre of the Universe (so to speak) had momentous consequences, as the Catholic Church well understood, consequences going well beyond the limited realms of elementary mathematics. Of course from a mechanical and physical point of view, there is a real difference, the laws of nature would be rather awkward, would we insist that the Earth is stationary and the universe is revolving around it[24]

Now to proceed to the fascinating problem of measuring astronomical distances, not only geometry is involved, although it is done stepwise. Using longer and longer baselines, each established on previous measuring principles, we are able to penetrate further and further into the universe. The baseline of the Earths orbit is used to measuring distances to close stars, but the trigonometric method goes even beyond that, if we can on good grounds assume that a certain collection of starts form an equidistant group moving in tandem, we can use the sustained motion of the sun over decades to involve baselines far longer than that provided by the Earth's orbit. To go even further, geometry is abandoned for entirely different methods. It was the momentous discovery of Ms Leavitt that there was a simple relation between the apparent magnitude of a Cepheid and its period, and as those could all be assumed to be about equidistant to the Earth as they were all concentrated in the same direction as the Small Magellan Cloud, one could find a new measuring device based on an assumed inductive law of nature[25]. Finding close by cepheids with known periods the scale could be calibrated and be used to measure inter-galactic distances. Thus we see a notion of length, differently interpreted because differently measured, but with some kind of over-lapping correlations. This is typical of the scientific enterprise, to extend our knowledge, not so much by straightforward empirical procedures, but by daring theoretical leaps. Of course such elaborate structures need to have internal consistency (some of which of course can be fudged) and be consonant with a hoist of other consequences. Ultimately our knowledge is so vast because the order of the universe seem somehow to be concordant with our own sense of order. And this is of course yet another meta-physical statement which has been made over and over again by different generations of philosophers and scientists. Now this brief sketch of boot-strapping is very typical of science, in which there are few direct observations, but any kind of observation builds on a long string of theoretical assumptions. This of course makes the notion of falsification subtle in practice, because there is essentially no such thing as an observation that is not theory-laden. Thus in science

this is a theory we find it awkward to maintain. But of course in certain situations this might be the most convenient way of dealing with perspective distortion.

[24] Of course for an observational astronomer struggling to keep his objects withing stable view, the idea that the universe is indeed revolving is a very palpable sensation.

[25] This is really remarkable as if a hand extended from God to assist the struggling astronomers to breach yet another leap in measuring distances

one has to live with (minor) contradictions, assuming that they are only temporary, and will eventually be ironed-out. After this digression let us return to the question of geometry, with which Carnap is duly fascinated.

It may well be that Kant was the last great philosopher and his legacy is of course pervasive in modern science[26]. Crucial to his view of the reasoning mind is the innate structures it endows outside reality with in order to make sense of it [27]. In particular he made two kinds of divisions of knowledge, that of *a priori* and that of *a posteriori* on one hand, and analytic as opposed to synthetic on the other. The first concerns the empistomology how do we know, innately or through experience, the second concerns the nature of our knowledge, whether formal and uncontestable (in a sense tautological) or whether empirical. The classic question Kant posed was whether there existed *a priori synthetic* knowledge, non-tautological knowledge that is nevertheless innate in us[28]. Now Carnap holds that Kant believed that Euclidean geometrical knowledge was of that nature, thus that something that related to physical space, but of which we did have intuitive knowledge predating experience. I have not (yet) read Kant, so I cannot make an authoritative response, but I would be a little more careful. After all does not Kant claim that Euclidean geometry is a matrix into which we arrange our spatial experience, and thus the notion that space is Euclidean becomes tautological (cf above the suggestion of Poincaré and the possibility of physical fudging to retain the Euclidean nature of space, come what may). Carnap makes a clear distinction between on one hand mathematical (and analytic) geometry and physical (and synthetic) on the other. Making such clear distinctions in order to evade ambiguities and confusion is of course the stock and trade of a philosopher, but I think the attempts to contrast a purely formal geometry on one hand, where objects have no interpretations, and a physical one is somewhat misleading. First our intuition of geometry is spatial, we have a notion of a point and a straight line, even if we can never realize them in nature (the closest we can come physically is the light ray), and thus those concepts become natural examples of Platonic objects. Secondly what excites us about geometry as a subject, especially the possibility of axiomatic geometry, is the power it gives us. The power not of a game of formal axioms, but the power over the outside world, which we seem to be able to conquer by thought alone[29]. The axioms involved are not just conventions, those are truths, self-evident maybe, but the more inevitable as such, and above all endowed with a moral force. One should also recall that the axioms of Euclid are of two types, one referred to as axioms, concerning rules of thought and as such analytic and logically binding, they correspond to the only way we are able and willing to think, once we would negate those principles, we would find ourself in a chaos out of which there would be no escape[30] The other is referred to as postulates, and concern the actual subject

[26] Kant himself was no mean scientist himself, his nebular hypothesis of the formation of the Solar System still elicits respect, although he was not able to make it as sophisticated as Laplace.

[27] There is of course he a suggestion, if not very serious, as to why this concordance between the universe and our mind might be tautological.

[28] Schlick elegantly characterised his own empiricist position as the impossibility of synthetic a priori

[29] And therein lies the excitement of rational reasoning in science, to impose our mind on the world, and thereby extending our knowledge well beyond the reaches of our limited sensory organs

[30] And recall even when we are thinking about logic as a subject matter, we adhere to those principles of

matter. In a way the uncontested postulates are in the nature of instrumentally bringing the nature of points and lines etc into existence (which can be employed for a purely logical game), while the contested parallel postulate (and its many equivalent forms) gives to geometry its non-trivial content (in particular its global reach).

Carnap is rather dismissive about mathematics, he refers to the successful efforts of Frege and Russell to reduce mathematics to logic, and ultimately to conceive of it as a string of tautologies. Mathematical truths are analytic truths, mere formalities which we impose on the world to manipulate it, but which do not say anything about the world at all. To a mathematician this is a travesty. Mathematics is in many ways an empirical science as well as a deductive one[31] most of the theorems of geometry are far from being obvious, we cannot say that they are true tautologically, that we cannot imagine the world in any other sense. We need to prove them, and finding proofs are not like calculations (although the checking of proofs maybe and some elements of proofs often are) but require ingenuity. Finding out a proof is like finding an empirical fact, the very proof itself is like an object in nature, the existence of which we cannot be sure of unless we have found it[32].

Now that notorious axiom resisted many attempts to be proved. The very fact that a deductive proof was felt needed suggests that the possibility of an alternate world. In fact the time-honoured proof by contradiction is nothing but the creation of an imaginary world eventually to be extinguished by the spark of a single contradiction. Many attempts at creating such alternative worlds were made, but rejected because of their perceived absurdity, such that there being intrinsic unit of lengths (Legendre) or that the areas of triangles were uniformly bounded. Absurdities maybe, but nothing impossible[33].

Now the way to think of say Hyperbolic geometry is not to think of a model of it, such as the Poincaré or Klein, which are nothing but conformal and gnomic projections, respectively, of the hyperbolic plane, useful as those may be for proving (relative) consistency and performing calculations, the real excitement comes when you place yourself inside it, assuming it has a physical reality[34]. Only then are you able to imaginatively enter into it and conceive of theorems that you would otherwise never have thought of.

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reasoning. We may conceive of strange logics, but in our meta-logic we do not stray from our accustomed principles.

[31] For one thing even if we think of it as a game we have the question of whether the axioms are consistent, this is an empirical question, and cannot be deductively inferred. By axiomatising mathematics, you make a physical object out of it, just as the implementation of mathematics on a computer is turning the spirit into flesh, and make its theoretical facts develop and manifest themselves physically. With the increased power of computers it has become more and more feasible to perform 'mathematical experiments' and this may also be, whether we like it or not, a feature of the future.

[32] This might suggest constructive mathematics, but the analogy is not meant to go so far.

[33] According to Coxeter Lewis Carroll rejected hyperbolic geometry out of hand because of the uniform bound on the areas of triangles. This is ever would be an example of a failure of imagination, ironically concerning a person whom by many is considered the most imaginative of mathematicians. Just imagine what could have been added to Alice in Wonderland with a little hyperbolic touch!

[34] Yes, I would love the universe to have a hyperbolic structure, there would be so much more space. Of course as Penrose has pointed out on the the three dimensional of world lines, there is a natural hyperbolic structure, making sense of the formalism of Einstein-Minkowski, and in which the photons make up the

Carnap points out that the Elliptic Geometry of Riemann came long after Lobachevsky. Why is that? Spherical geometry has been part of mathematics since antiquity, why did not anyone think of that? Obviously you do not need any new geometry to understand the sphere, it sits there embedded in 3-dimensional space. Furthermore great circles are curved and are no straight lines, and in addition they meet at two points unlike lines (to take the anti-podal quotient is not a natural thing to do at all)[35]. On the other hand I claim that spherical geometry is the one with which we are most familiar, not as seen from the outside, but more naturally from the inside. Our field of vision is a sphere, something which becomes palpable when we look at the celestial sky. Straight lines are of course intersections with planes through the eyes of the observer, and thus great circles when viewed from the centre look straight. Although the gnomic projection is not conformal, we do not use it. When we look at two lines intersecting, we look directly (not askew) at the point of intersection and hence the angle coincide with the spherical one. Also, we have no problem conceiving of this bounded 2-dimensional continuum without a boundary, for some reason we do not miss any lines of directions: but if we radially extend our sphere to infinity, it is hard for us to believe that those radial lines will not go on for ever. Why is that?

Non-Euclidean geometry is a delightful subject, far more pleasing to the mathematical mind than its tangential approximation. While in Euclidean space you may parallel transport a rod along a curve and it will return to itself, but in non-Euclidean space, there will be a discrepancy (as surprising as if the rod had changed length) the bigger the larger area the curve would encompass. This ties up with the notion of curvature and hence gravitational attraction, thus making Einsteins general theory of relativity so pleasing to a mathematician.

Now Carnap brings up the question whether Gauss actually did measure the angles of triangles to empirically test our geometry. If it was done in the belief that the universe was homogeneous, i.e. of constant curvature, those attempts would have been pointless. If the natural unit would be of the size of the Earths radius, the effect on any triangle of the type Gauss considered would have been beyond the accuracy of measurement, on the other hand the astronomical consequences would have been momentous, there would have been a marked parallax of the starry sky resulting from rather modest terrestrial movements, and the sun would never be visible in the Arctic regions[36]. Clearly Gauss thought of this, as I believe he thought deeper (and quicker) about hyperbolic geometry than any other, and must have been familiar with those effects. On the other hand he might have gone one step beyond conceiving of the universe as of drastically varying curvature.

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boundary firmament at infinity, and which the phenomenon of abberation is simply hyperbolic parallax, which is present even for infinitely distant objects.

[35] I remember myself being puzzled as a teenager by all this fuss about alternate geometry, there was Euclidean that explained everything about the sphere. Of course no similar embedded model can be made of the hyperbolic space, unlike in hyperbolic space there are similar models of both Euclidean (horispheres) and Spherical geometry (ordinary spheres).

[36] Lobachevsky was very much concerned with hyperbolic, or astral geometry as he called it, as a physical possibility. The absence of parallax at the time, it was not until 1837 that Bessel would measure the first parallax, indicated that the natural unit of hyperbolic space must be quite large

The preceding sections may have conveyed something of the excitement of power that a geometer may experience in identifying (against the recommendations of Carnap) his geometrical vision with physical space 'out there'. It is by virtue of his geometrical intuition he can imaginatively transpose himself to distant parts, and it was indeed the genius of Newton to assume that terrestrial conditions also reigned in the celestial sphere, by extending the laws of nature to an infinite realm. Why is this possible? Because as we have noted before of the concordance with our sense of order with that of the universe, a sense that is not just imposed (as certain aspects of it can be) but so far provisionally confirmed. Now the real excitement of the basic sciences is not just that we can state empirical laws of supposedly universal application, but come up with theoretical constructions, which generate a hoist of empirical laws we would not otherwise be able to think of[37]. The creations of those theoretical constructs are like all creations basically enigmatic but their creations are abetted by the constraints of empirical facts on which they are bound to come into contact[38]. A theoretical construct involves entities which are not observable, in order to make sense of a theoretical construct, which in a way can be thought of a Platonic entity which needs to be projected down to our sensory world, there need to be what Carnap conceives as correspondences, some kind of interpretation of their effects. The objects that make up a theoretical construct may not really exist (at one time atoms were considered to be convenient entities without any real existence[39]) they may be just convenient virtual variables so to speak to ease calculation. What is an electron I used to wonder? A particle or a wave or both, or simply an electron, knowable only instrumentally? Those unobservable hypothetical entities are somewhat of an embarrassment and thus Carnap discusses at length the stratagem of Ramsey to free descriptions entirely of their explicit presence. This of course ties in with Carnap's overriding ambition to create a precise and formal language, maybe similar to that of Russell and Whitehead, an ambition I believe has been abandoned[40]

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[37] The acceptance of Einsteins theory of gravitation is not only based on its great aesthetic appeal, but to the new phenomena it predicted, and which were, as noted above, confirmed. On the other hand beautiful as it is, the practical applications of general relativity are modest, the implications of Maxwell's equations (incidentally pointing to Einstein through its invariance under Lorentz transformations) turned out to be far more momentous in transforming not only physics but everyday life in the 20th century. Now practical applications are not the goals of science, they merely indicate that something real and non-trivial is going on, and provide further confirmations of theories as well as new ways of testing them.

[38] Popper points out that it does not matter how a theoretical construct is thought of, what matters is to what extent it is compatible with everything else, and restrictions and constraints stimulate the imagination more than anything else, this is why censorship paradoxically is not necessarily always such a bad thing.

[39] I recall being puzzled as a child being told of the Wilson cloud chamber and those streaks which my father as a high-school teacher was introducing in the experimental curriculum. Were those single tracks really the track of a single individual atom, or were they just some kind of statistical coincidence? It seemed strange that out literally trillions and trillions of atoms, to single out an individual. This is of course the mystery of a micro-phenomenon being manifested on the macro level.

[40] Similarly in the 20's and 30's there were ambitions of creating a new kind of logic to more effectively deal with the phenomena on the quantum level. Cf. my remarks about the difference between logic and

Finally as to the question of a free will. Carnap does not think this is a real problem, after all without some measure of determinism we cannot make free choices as we cannot reliably predict the consequences of such. Furthermore that there is a distinction between following your inclinations and being under compulsion to do so. On the other hand I think he has not given the matter sufficient thought. Imagine the universe to be deterministic, that could mean that we are conditioned to believe in free will. All our actions flow like following a gradient, yet with our conviction that we are just following our inclinations (as we literally are in fact). If there would be no free-will we probably would be completely unaware of it. Also all our truths that we find would be contingent upon predetermined facts and as such not necessarily congruent with truth, only forced on us by our reasoning yet (supposedly) useful and inevitable conventions. Once again we are encountering the age-old problem of including the observer into what is being observed.

Now what about going back in time? Einsteins theory of general relativity is like the philosophy of Parmenides, it just sits there timeless[41]. But there is no contradiction in time-travel as long as there is no free will. But of course with free will you may mischievously undercut what in the first case made it possible, such as killing your mother when she was a girl. This is of course the Cantor diagonal trick again, and this is why I refer to it as a manifestation of free will.

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meta-logic. I agree that such ambitions were futile, there is mathematics for manipulation and discovery, and the ordinary human language which may not be formally manipulated but has the great advantage unlike formal artificial languages of being its own meta-language.

[41] This bothered Popper, and stupidly so I would say