Coxeter

King of Infinite Space

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As a young teenager I came across the Platonic Solids in a collection of Martin Gardner. I was immediately taken, and started to make card-board models and started dreaming about finding the 4-dimensional analogues, no doubt inspired by the hypercube I had encountered in Gamows 1,2,3 infinity earlier that year. I recall that I tried by reasoning through analogy, which invariably led me astray, and the ambition remained a mere dream and was eventually given up, yet the more general ambition it sparked, that of becoming a mathematician, proved more enduring. At the time I had no idea of the name of Coxeter, let alone his book on Polytopes, which no doubt would have been beyond me at that stage. Strange as the ambition may have been locally, the great popularity of 'Regular Polytopes' testifies to its global appeal, and it certainly lacked no precedents, Coxeter being of course the supreme example. Fortunate indeed is the individual who can base his life on a childhood preoccupation, pursue it with the simplicity and purity that is the golden hallmark of a child, into extreme old age, with no lessening of lucidity and commitment.

However, lauded as Coxeter is nowadays, this appreciation was not always apparent. Why this is, is an interesting and illuminating question to pose, and one which the author addresses in her biography. The author is no mathematician, although unlike so many of those 'out there', endowed with an appreciative attitude (on the other hand, why would she had bothered to write the book if not?). She has interviewed a large number of distinguished mathematicians and synthesized their responses. The result is invariably marred by a certain lack of proportion as well as journalistic hyperbole (too call 'Regular Polytopes' the geometrical counterpart of Darwins 'The Origins of the Species' is admittedly to stretch things a bit far.) This is a pity. On the other hand the book is not so much intended for the professional mathematician, although I suspect those will belong to the most appreciative readers, as for the general public, with the dual purpose of trying to make a representative of that esoteric entity - the pure mathematician, come alive, as well as whet the appetite for exactly what made the great man 'tick'.

The problem with Coxeter as a subject of biography is that the life he led lacked in external excitement. He married the first woman he met, conducted an unremarkable family-life (his spouse and children occasionally expressing their frustation at his aloofness), had no mistresses (except of course mathematics), and did in no way participate in the public adventures of the 20th century. True the beginnings had the charms of Victorian life surviving past its expiration date, with touches of Alice in the Wonderland and the idyllic serenity of Cambridge College residence, providing a haven for the shy young boy of divorced parents to devote himself entirely to his calling. He did have dealings with both Hardy and Littlewood, although he studied under the nowadays mostly forgotten geometer Baker, and he even crossed paths with Wittgenstein, although finding his philos-

ophy incomprehensible. Thus with his definitive settling down to Toronto in the 40's the biographical narrative basically comes to an end, save for the intermittent snippet. The focus on the book is on his status as a geometer, the King of Infinite space, as the title announces, and the saviour of the same, as the subtitle explains.

Modern mathematics of the 20th century is a forbidding enterprise, requiring extended technical expertise from its practioneers, confining most of them to a narrow groove, their competetive edge dependant on their ability to exploit high-technology results of which they may have only superficial understanding. The idea of complete self-sufficiency pursuing simple objects in depths must strike most of us nowadays as an almost impossible ideal. Coxeter proves the exception to that curse. His career was devoid of competition (which did not prevent him to savour the prizes that came in his way later in life) neglecting contemporary trends, focusing sharply on what really gripped his imagination. The point of departure so close to being dismissed as mere recreational mathematics, but when pursued with tenacity yielding insights far transcending the seemingly humble beginnings. In the book Coxeters brand of mathematics in general and geometry in particular is contrasted with the formalism of Hilbert and the school of Bourbaki. Personally I find the view relayed by the author as somewhat misleading although it is well-spread. It is true that Hilbert introduced formalism into geometry, but the purpose of Hilberts formalism was instrumental for one particular purpose, namely that of proving logical consistency, and did not reflect a personal view of mathematics. In fact once could liken his ambition to that of representing pictures pixel by pixel, greatly faciliating their reproduction, but in no way involving their appreciation. Also I believe that the influence of Bourbaki, in particular the supposedly malignant one, to modern mathematics has been exaggarated, many active branches of mathematics having been untouched. It is true that it emphasized rigour and precision, and that may have inspired some educators to implement pedagogically disastrous methods in elementary math education, resulting in the enduring misconception of viewing professional mathematicians as formal Bourbakists and thereby barring them from future meddling into schools. Dieudonne, with his cry of death to triangles, is set up as the opponent to Coxeter, but the irony must not have escaped most mathematicians, that the concluding volumes (and some of the most successful to boot) of Bourbaki were devoted to Coxeter systems, eminently presentable in their particular style of exposition.

The mathematics of Coxeter allows a very formal and economical encoding, but to view it as a frivolous play on simple rules is deeply misleading. The apparent simplicity is the result of distillation, and the play is not frivolous but serious like the play of the child. The underlying appeal goes beyond the combinatorical and enters into the Platonic realm, testifying that some mathematical objects are indeed more central than others. Ultimately it is of course about symmetry, that abstract notion manifested to us by the projections it casts into the visual and tangible world. To the serious mathematician the objects of mathematics are imbued with deep meaning, without this sense he may be technically brilliant but he will be unable to push beyond. It is my belief that Coxeter must have been obsessed with this conviction, otherwise he would not have been able to push beyond the merely curious. Some people say that we mathematicians create our worlds, but if so, we, unlike the poor painters, are able to realise the dreams of entering into our creations. The multi-dimensional geometric worlds, as well as their alternates, may be closed to our

senses, but not to our intellect and imagination.

Visual artistry and mathematics is sometimes connected, although not as commonly as music and mathematics, and Coxeter is often brought up as a case in point, although his involvement to music was more relevant than to visual art for which he apparently showed no remarkable interest, at least not of the eclectic kind. Much is made of his mother being an artist and his father an amatuere sculptor, which nevertheless may have had some decisive influence on his temperament as opposed to its object. Publicly he is also connected with the names of Buckminster Fuller and Escher. Of the former the less said maybe the better. Coxeter eventually took exception to appropribational instincts of Fuller (the author characteristically digresses on Fullerenes, a molecular combination of 60 carbon atoms, forming the vertices of a truncated icosahedron of twelve pentagons and twenty hexagons known since antiquity and familar to all soccer players) while the relations with the Dutch print-maker turned out to be far more congenial. In the world of Art Escher remains a marginal figure, while being a favourite of mathematicians. Being an autodidact he provides one of the few examples of mathematical innocent exploring visual mathematical themes and actually coming up with appropriate solutions. Coxeter repeatedly tried to educate him by standard symbolic explanations, and in spite of their, at least to the professional, elementary nature, they proved opaque to him. This opens up an intriguing avenue of exploration concerning mathematical intuition and visual manifestation, which, maybe wisely, the author does not pursue in depth.

As to the lasting legacy of Coxeter he undeniably will hold his place in 20th century mathematical history, but he is not the towering figure, not even in geometry, who the author may try to make him out to be. His contributions are not in doubt, but without the wider mathematical culture they come embedded into, their significance would not emerge fully. Coxeters way of doing mathematics provides an inspiration, but of course it is not the only one. Mathematics, in spite of the occasionally truly towering genius is a collective enterprise, which no individual any longer can expect to master in its entirety. High-technology mathematics has come to stay, and although there is still space for Alice in the Wonderland, the Wonderland is now much larger than Alice can expect to explore.

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