Descartes' Dream

The world according to mathematics

P.J.Davis and R.Hersh

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This book is not about mathematics per se, but mathematization, i.e. the imposition of mathematics on the world, especially through rampant computerization, still in its infancy back in 1986, but yet already pervasive. Mathematics is one thing, mathematization quite another. Mathematics, according to Hersh (clearly the major author of the book, interspersed with a few interviews conducted by Davis), is a wonderful creation of the human mind, having far reaching applications, but it is just one part of the human mind, and so much that is dear to us lies beyond it, and the forceful imposition of it, almost never conducted by mathematicians, is constrictive and distortive.

Hersh is a humanist, seeing mathematics on par with religion, music, literature, and as such subject to the ravages of time and fashion, and far away from the cold rigid constraints imposed by Platonism. The mathematization of the world, the vision entertained by the likes of Descartes and Leibniz in order to make the world managable, is so often inappropriate. There is a hierarchy of sciences, with mathematics seens as the ultimate and most basic, hence the misdirected ambition of trying to quantify as much as possible of scientific inquiry, to make it respectable, because after all what is more objective than number? True, what is more objective than number, but it is not number per se which is the problem, but its relevance in new and often alien contexts. Mathematics is often ridiculed as being too crude, too misleading, but of course it is not mathematics which is at fault, only its positioning. Too many of the important facets of human existence cannot be reduced to number and number crunching, in fact number crunching itself cannot be reduced to mere crunching of numbers, even in numerical analysis, the straightforward computation is not straightforward but depends on personal judgement and sense, and cannot be fully algorithmized and computer packaged, thus keeping numerical analysists in business.

Statistics is what most people think of when they think of mathematics. Statistics, as most people conceive of it, is not mathematics but mathematization. The reduction of complex realities to strings of numbers to be manipulated. There is nothing wrong with the mathematical manipulation per se, what is problematic is the huge leap that is taken from probability theory, a branch of pure mathematics, via applied statistics to the real world. What does a probability really mean when interpreted in a real world situation? This is a non-mathematical problem, on which the mathematical analysis cannot shed much light.

A large part of the book is devoted to computerization. Computers are part of the real world as they are not just abstract programs. Computers tie up with money and technology, technology makes for new soft-ware possibility, which in their turn can improve technology and hence computers themselves, leading to an un-ending spiral of develope-

ment and improvement. Can this continue indefinitely? The speed of light and the finite sizes of elementary particles put some definite bounds on the powers of computers, bounds that have not yet been reached, but seem almost reachable. This means that the physical world actually sets a bound on how extensive computations can actually be. In mathematics there are so many questions you can ask which will for ever be beyond the capabilities of computers. Is this a comfort or a source of frustration? Depends on the context, so yes and no.

The programmer is a new animal on the human scene. Who is the programmer really? Someone who feels more comfortable with machines than with people. This is a disparaging characterization if any, but probably quite true. What is the kind of thinking that goes into programming? It is of two complementary kinds. One to break down things into the small steps, another to build up from modular units. Going down and going up. Somebody who does not like to do cross-word puzzles would probably not be a good programmer. Maybe true, yet as misleading as to say that a person who is not good at chess is not good at mathematics. Are mathematicians good programmers? Probably not, they would too easily get bored. Maybe, maybe not. The fact is that programming is seductive to many mathematicians, it is not a mathematical activity, but mathematics itself provides both the powerful means as well as an even more powerful incentive to do programming. It is seductive, as you never get stuck, there is always something to do, and you always feel that you are at the very end of completion. Programming means interaction, real interaction for a mathematician, even if that interaction is only with a machine, it is concrete interaction nevertheless, a sequence of clear cut falsifiable steps which are tested one by one as you go along. You always do mistakes, but in real life mistakes often go unnoticed and cause no particular harm, while in programming every single little mistake makes the whole machine come to a schreeching halt. Of course there are other kinds of mistakes, meta-mistakes which have no direct bearing of the running inside the machine, only on intentions. The machine knows not of intentions, it is concerned with the repetitious execution of meaningless steps.

What kind of people are mathematicians? They certainly are not as stereo-typable as mere programmers, but certainly they do have their blinkers. Hersh does not like the making of lists, but he cannot refrain to make a list of features to be found in a mathematician. Mathematicians are good at many things. Abstraction, isolating the crucial features, the ability to make clear distinctions. Such features make for lucidity of thought, and with lucidity there comes power. But every medal has another side, and abilities that endow power in certain situations, are postively hampering in different kinds of contexts. Hersh makes some half-hearted suggestions that mathematicians should be more widely educated, that they should learn about history say. Such well-meaning exhortions are like ineffectual blows in the air, fuelled by laudable motivations they hit no target and dissipate their energy fruitlessly. The impulse to know is an inner one, it is not induced externally. Mathematicians who have the right temperament will invariably seek out that larger education by themselves, while those who lack the need, may go through the motions formally, but will remain as innocent as they were initially.

Should people learn mathematics? How much mathematics is really needed for the average person? In former days most peoople could not count, and the skills of performing

the four arithmetical operations were not widely distributed. Then it became part of compulsory education, and everyone was expected to be able to do long division. Nowadays most educated people are supposed to know calculus. Most people have no innate desire to learn, and even less have the need to knowcalculus, which hence reduces to a chore. So why is calculus so widely taught? To the mathematical community it is a godsend, however much they gripe about it, because it means money, more jobs, more opportunities for mathematicians to do what they like and get paid for it to boot. But from the view of society at large, is this such a desirable state of affairs? There is a rationale for it. Mathematics acts as a filter. It makes the flow of applicants to various institutions of higher learning more managable. Mathematicians act as gatekeepers, they will take the flack, while those institutions who have instigated the requirements get off scotch-free. Mathematics is a small field, yet the importance and inevitability of mathematics is toted more and more.

The book is written in the 80's, the cold war is still going on, and concomitant with this the threat of mutual assured annhilation. This surely is the overwhelming threat (nowadays having been supplemented by terrorism and global warming, threats that do not seem as catastrophic as a nuclear holocaust) and all problems of mankind are clearly subservient to this, the most acute and devastating one. Mathematics is a wonderful thing, a beautiful creation of the human mind, but beautiful as it may be, is it ultimately good? If it does play a crucial role in the destruction of us all, it clearly cannot be. Those concerns are deep ones, and they provide the penultimate essay in the collection.

July 19, 2007 **Ulf Persson:** *Prof.em, Chalmers U.of Tech., Göteborg Sweden* ulfp@chalmers.se