Development of Mathematics

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When I was fourteen I read Bell's 'Men of Mathematics' and it had a profound effect on me and my choice of career. The great mathematicians became my heroes and role models for better of for worse. Bell's book has been critized on many grounds as to historical accuracy and he has been accussed of merely presenting Hollywood scripts to pander to readers. Still, as Hardy said of a book that had imprinted him early in life, a book that fires the imagination of a young person cannot be that bad.

'Development of Mathematics' is written in a similar vein to 'Men of Mathematics' but with more emphasis on the mathematics. It is doubtful whether it would have had the same impact on me as the previous book had I read it as a young man. The deficiencies of Bell, as a writer and a mathematician, come to the fore in this work. It is written by an opinionated man, which of course adds to readability if not reliability. His greatest ire he vents against Plato and Platonism in mathematics. As many philosophizing mathematicians he sees it as antiquated superstition that has stifled mathematics for far too long and ought to share the fate of astrology, namely permanent abolishment. Maybe its only eternal aspect? On the other hand he does mention in passing that Platonism is about mathematical realism, and without some faith in realism when mathematics is concerned, his two overriding messages about mathematical progress and development - increased rigour and abstraction, do not make sense. He writes approvingly of the deductive method of mathematics, without which there will be none at all, just guesses founded on nothing but whim and wishful thinking. And the increased level of abstraction, which in particular algebra has manifested, what is that if anything else but a disclosure of underlying forms that explain the embarrassing variety and overwhelming confusion of special cases. Platonism is mostly taken too literally, one should not blind oneself to the actual historical manifestation of Platonism as it has been conveyed by Plato and his followers. It is true that his personal opinions of mathematics tended to rigidify and constrain it unnecessarily, but that one can safely ignore, maybe not as a historian of Classical thought, but as a living philosopher.

Bell does not spend much time on early mathematics, but concentrates on the 18th and 19th century, and with the hindsight the first half of the 20th provides. The second edition, which I am reading, stems from 1945, and the predictions he makes about the year 2000, which must have loomed impossibly distant at that time, are risably off the mark. To assume that present day mathematics may be as outmoded in the next Millennium as Babylonian mathematics was at the war, belies a certain lack of sound judgment. The argument is based on the rapid development of science, but to say what before took fourthosuand years is now dispensed with in a decade is a hyperbole of only rhetorical value if any. He observes reasonably enough that one should be wary of reading in too much in the works of tha ancients. It is of course easy with good will to detect various anticipations of modern ideas in the work of the ancient, but one should not forget that only hindsight allows such conclusions, at the time of writing it is far from clear that they understood what they were at the verge at. Also, what counts is influence, to detect a fully developed calculus say, on some ancient cuneiform tablet, is a case of curiosity as far as the history of mathematics is concerned. Connected to this is the convention of first publication. The rules are unforgiving Bell points out, yet the procedure is of course a bit too rigid. If we want to have a 1-1 correspondence between deed and individual, this might be as good as any other arbitrary method, but it does not properly give due to effort and ingenuity, and as far as history is concerned, actual influence will count for most in the eyes of posterity. And execeotions to the rule are rampant. Lobachevsky is traditionally recognized as the founder of hyperbolic geometry, yet the claims by Gauss and Bolyai are not ignored. In fact there seems to be a general understanding that any significant advance of 19th century mathematics, first germinated in the mind of Gauss.

The book is thick and thus one expects it to cover a lot of ground. And indeed most mathematicians of worth are mentioned somewhere in the book. The presentation is not chronological, but thematical, with delineations of geometry, algebra and analysis. When it comes to geometry Bell is an unabashed stickler for analytic geometry, which he considers to have enlarged its scoop tremendously and considers so called synthetic geometry, as practised and propagated by Poncelet, something of a curiosity. As to the abstraction of algebra he is impressed and clearly thinks that this provides a model for mathematical development, even if he thinks that much of what has been done will be forgotten. As an example he takes all classical work in invariant theory as represented by Gordan became obsolete by Hilberts approach. (Twice does he quote Gordans famous remark of mathematics versus theology). In general, concrete and intricate examples, as were often worked out in the 19th century, he dismisses as antiquarian museum pieces, and as an exemplary case he brings up the horror of the twenty-seven lines on the cubic surface and its various symmetries and double-sixes and what not. It is probably true that at the time such type of mathematics was at its lowest ebb. More to the taste of Bell is topology, which he finds awesomely abstract, and with admiration he points out how topology will unify geometry and analysis.

Some parts are devoted to rigor in analysis, marvelling at the sure-footed intuition of an Euler allowing him daring formal manipulations and still landing on his feet. But his analysis here of the concept of real numbers and limits and continuity suffers from a lack of precision. Bell does not allow himself to get bogged down with details, which makes his analysis superficial and in the end unsatisfactory. A large part is devoted to applied mathematics, how physical problems have actually inspired mathematics. What is interesting here is the clash between physical intuition and mathematical rigor. That solutions exist to certain differential equations is obvious from the phyisical point of view, but mathematicians can cook up counter-examples. What does that mean really? One respons would be that the mathematical world cooks up situations that have little relevance to the much cruder physical one. In what sense does the uncountability of the reals have a physical meaning, in particular as to peculiar sets, such as the Canto set. Metaphorical definitely, with all its fecund suggestions, but mathematics takes things literally, and suggestions lead to implications which are carried to the extreme. Deduction compels, and the mathematician are led to the end of the rope, while induction merely permits, if provisionally and pragmatically. As already noted the issue of rigor in its most unforgiving aspects does only make sense if there is a seperate mathematical reality, of which the physical world is just a blurred image. To make a separation between mathematics and the physical world leads to problems akin to those of Cartesian dualism. One way of dealing with it is to reduce mathematics to the level of a language.

Bell is not unaware of the battle of the frogs, namely the controversies concerning the foundations of mathematics. As mathematics became more rigorous and abstract, basic logic appeared to play a more and more significant role. As ot did some basic inconsistencies evolved from careless thinking, and the stratagems to circumvent them lead to complications. Brouwer proposed that the excluded middle should be abolished from mathematics, the metaphysical idea that a statement be either true or false, leading to much monstrosity, in particular the calling into existence strange concepts by a method reminiscent of scholastic theology. Instead existence should not be a matter of mystical invokement but a result of honest and painful construction. And so the school of intuitionism, with roots of earlker 19th century attitudes such as that of Kronecker, was born. It heralded a tradition of constructive mathematics, which became particularly relevant with the advance of high-powered computers, which still lay in the future during the writing of the book. Bell also mentions Gödel but does not delve into his proof except very superficially.

At the time of Bell's writing mathematics was more productive than it had ever been before, which would hold to be true for decades to come, implying that the level at the time was still relatively modest. This excessive production of mathematics by armies of mathematicians cannot but make an overwhelming impression on the onlooker. How much of this will be of enduring value, how much of this is not merely the mechanical output of second-rate mathematicians, pressed by circumstances as well as idle curiosity to investigate any possible avenue? Of course the situation in big sciences is far worse, there the individual has little capacity for a serious overview and is inevitably constrained into ever narrowing loads of quarry.

I have touched on the opinionated style of Bell. He is also sarcastic. As to the applications of mathematical elastic-solid theory, he refers to their great range, from designing earthquake-proof buildings to long-range heavy artillery for reducing the same to piles of rubbish. He notes that war has accelerated progress in mathematics (and science). Without the First World War civil aviation as we know of it would have been severely retarded (and not a bad thing I would say). The reason being that in war, personal taste and pace, will have to step down for collective imperatives. Things which normally would have been considered too boring, now have to be faced head on. As an example he brings up non-linear differential equations. He notes sarcastically that optimistic mathematicians once prophesied that Russells and Whiteheads *Principia mathematica* would greatly assist judges of the U.S. Supreme Court. As to the blessings of abstraction he notes that it will greatly facilitate the learning of mathematics and significantly lessen the strain on memory, and that present day dissenters only maintain that the proper way of imparting mathematics is to make it as tedious and repulsive as possible. If mathematics is to progress, Bell informs the reader, it cannot encumber itself with antiquated baggage. He concludes with the prophecy that by the year 2000, the last adherent of the Platonic ideal in mathematics will have joined the dinosaurs.

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