The equations

icons of knowledge

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Equations are anathema in any popular book on Science. Reportedly the publisher of Hawkins 'the Beginning of Time' warned the author that everytime an equation would appear on the page, half the remaining readers would drop out. Hawkins popular book contains no equations, except of course the ultimate icon $E = mc^2$ and as a result is, I suspect, as incomprehensible to the expert as to the layman. The author of the present book, the Dutch theoretical physicist Sander Bais, takes the opposite tack. Equations are to be given center stage. How could one ever think of a book on art history without a single picture, writes Bais and likens his own effort as a kind of picture gallery in which the reader is invited to sample the equations in no particular order. The result is a so called coffee-table book, of slim proportions with very little text on each page, and each equation, or sets thereof, generously displayed on single pages. What possibly could anyone learn or profit from this?

Mathematics is the language of nature, a saying that goes back to Galileo and ever since retaining a lot of weight. One should, however, see it as a metaphor, and the point of a metaphor is to evoke, not to instruct, and thus should never be taken literally. But the metaphor of mathematics as a language persists and leads to much misunderstanding and misdirected efforts. One of the consequences is that mathematical formulas are seen to be expressions in a strange language, and as all strange languages amendable to translation obviating the efforts of learning the language. The problem is that there exist no translations. Of course we can say that the gravitational force between two bodies is proportionally to each mass separately and inversely proportional to the distance squared instead of writing $F = \frac{Gm_1m_2}{r^2}$ but that is a trivial translation. When mathematicians and physicists manipulate formulas they are not engaged in expressing themselves in language, as little as we are expressing ourselves in language when we fiddle around in the garden, or tinker with the car. The problems of mathematics are not the problems of language acquisition. One can of course describe equations and talk of them, but that is no substitute for dealing with them. Equations are also sometimes likened to poems. An incredible amount of information condensed into very little space. Once again we are dealing with a metaphor, or merely a variation and specification of Galileis, and as such directly misleading when taken on its face value. Of course an equation can be beautiful and mysterious, but the beauty, even if sometimes reflected in its typography, cannot be divorced from its meaning, and to find out the meaning of an equation is quite a different thing than analysing a poem. Poetry can be thought of as a tension between sense and sound and the ultimate realisation that the one cannot be separated from the other. Equations have no sound to start out with.

So whatever can be conveyed about an equation to the layman? Any writer on popular mathematics or physics has to suppress his basic instinct of systematic instruction. Still there are always things that can be discussed without any technical familiarity on the part of the reader. A mathematician and physicist do not always manipulate blindly equations, they also step back and think and reflect upon what they are doing. Motivations play a crucial role in all activities, and the laying bare of motivations may be the most valuable a writer on popular science may do[1]. To the most amendable examples belong the considerations of relativity theory, expressible as so called thought-experiments. It is of course deeply misunderstood by the public, in ways that are not inevitable. Relativity theory is thought to be about relativity when it is in fact about invariance. The absolute invariance of the speed of light. It says that time and position depends on the observer, hence the relativity of being, so exploited by fuzzy-minded post-modernistic thinkers. But nevertheless the fundamental laws of physics are the same for systems in uniform motion with respect to each other. Neither more nor less. If you have two systems in acceleration with respect to each other, the laws will differ. One striking illustration of this is the light-ray in a freely falling elevator. From the point of view of the observer in the elevator the light-ray does not follow a straight line, but is deflected as if affected by gravitation. Such musings do require very little technical apparatus, only some clear thinking, and should thus be accessible to the intelligent layman. The author brings up the so called twin-paradox, without pointing out why it seems to be a paradox and why it actually is not. It hinges on the fact that times differ in moving systems[2]. Thus if one twin goes off at a sizeable fraction of the speed of light and then returns he will have aged less than his stationary. The paradox is of course not that one twin has aged less then the other (this happens all the time in real life) but on the fact that the situation appears symmetrical. Can one not think of the roles interchanged? Relativity is after all about the absence of an absolute space, and questions of whom is rotating about whom are meaningless unless an arbitrary point of reference is chosen. In particular the question of whether the sun rotates around the earth or vice versa seems to be, like the old catholic theologicians claimed a matter of mathematical convention[3]. This is simply not true. The two sets of references, one for each twin, are not in uniform motion with respect to each other (if so the other twin would never have been able to return to the first), thus they can be intrinsically distinguished. In more technical language. The worldline of the stationary twin is a geodesic in the Lorenzian space-time manifold, while the world-line of the other is not, and unlike the case of ordinary Riemannian manifolds geodesics locally maximize distances rather then minimizing them. In particular in Einsteins theory once can actually say that the Earth rotates around the sun and not vice versa, even if there would be no other matter in the universe.

[1] And I am not thinking here of stupid practical applications, who cares about them anyway? They may admittedly bring some motivation, just like money may motivate some people to work. But the real motivation in any intellectual work is intellectual in nature

[2] Strikingly experimentally verified in the documentation of rapidly decaying elements in high-speed cosmic rays

[3] Or more precisely they had no objections in principle against treating the Copernican model as a mathematical tool for simplifying calculations, but to really say that the sun, not the earth was the center of the universe was heresy

Equations contain a lot, literally far more than meets the eye. Their value lie not so much in their predicative power (i.e. allowing numerical simulations on the computer) as on their conceptual instructive one. There is much talk about modelling. Mathematics providing models to simulate reality. This I find a rather sterile point of view, although it does have many practical applications. A good equation contains much more than was ever put into it. This is at the core of the mystery of scientific inquiry. There are many more aspects to an equation than the relations it encodes. Symmetries can sometimes be obvious to the naked eye, but often they are hidden, like the Lorenz transformations that leave Maxwells equations invariant. The finite propagation of light is a consequence of them, and its symmetries point towards special relativity long before its originator was even born. Dirac is famously known to have claimed that he was more guided by mathematical beauty than empirical confirmation. Eventually he was vindicated. String theory, the latest development, with the overarching ambition to include everything, has no resource to experimental testing, only mathematical coherence and beauty, the ultimate hope being that it will emerge by necessity as the only feasible theory. For this reason it is still viewed with suspicion by many practical physicists, and so far it has had no applications to physics, but on the other hand many to mathematics.

A book like the present is bound to be fragmentary, in fact this is its very ambition, yet it does present a few unifying threads, the most obvious one being that a string of equations can be used as landmarks to charter the progress of modern physics since its inception by Newton. To take one example - the Schrdinger equation, the archetypical equation of the emerging Quantum theory. Its sources harp back to the classical equation for the harmonic oscillator, with which it shares a few formal properties. It was however not consistent with special relativity, and for the next step comes Dirac combining the two in his equation for the relativistic electron. An equation that actually predicted (and once again we are not talking about simple modeling and numerical simulations, but conceptional insights having to do more with the structure and 'hidden agendas' of equations) the existence of a positively charged electron - a so called positron, and ultimately the notion of anti-matter in general. Thus equations lead to new unexpected insights independant of empirical observations, but of course subject to their verifications[4].

In a book with such an ambition and with such severe constraints it would be pointless to look for omissions. Omissions are inevitable, each new author inspired to treat the material in a totally different way. The idea and the ambition is a good one, maybe even an excellent one, but it is hard to bring to fruition. One may argue with the choices of equations, some of those are canonical, especially Einsteins refered to initially, if ever a true icon[5], others more peripheral, like Korteweg-de Vries equation. The author is of course welcome to make his own choice, but some explanation would not be amiss in the case

[4] To be somewhat pedantic one should perhaps instead talk about the falsification in the spirit of Popper

[5] Formulas can be expressed in many different ways, using different letters for the variables, and rearranged depending on purpose, mathematicians do that every day. But Einsteins famous equation displaying the equivalence of matter and energy, can only be expressed in those letters, this is after all what is meant by being an icon.

of its inclusion[6]. The question of what is meant by a solution is an important one when equations are concerned, after all equations are there to be solved. But it is not so clear what is meant by a solution. Traditionally one distinguishes between solutions in closed form, i.e. explicit combinations out of a limited library of standard functions (for most laymen and mathematicians alike limited to elementary functions like exponentials and trigonometric functions and their inverses) or numerical ones. The former only making up a tiny minority. The celebrated three-body problem bequested to us from the pioneers of Celestial Mechanics admits of no closed form and in fact the longrange behaviour is unpredictable, thus showing that chaos can be totally deterministic, something pointed out already by Poincaré and in recent years popularized as the so called Butterfly effect. In fact solving equations is only part of their study, not necessarily the most important. Equations are there to suggest things. One may argue that the notion of the conversation of energy was suggested from equations and as the notion enegry became refined its conversation was a leading principle. In the end we needed to include in energy the hidden one 'frozen' in matter as well. One may argue that this is circular reasoning, in a strict sense it may be, but nevertheless it points to a fruitful way of organizing thinking. Another notion that pops up from equations is the one of entropy. It is not clear how to interpret it, the usual explanation of 'disorder' being not very instructive. This illustrates that the real difficulty is not so much to 'translate' into mathematics (to use the point of view deplored above) but to give everyday intuitive meanings of its emergent features. Thermodynamics is also another area of classical (i.e. 19th century physics) which is easy to present to a lay audience with many everyday applications to support the intuition, with its hierachy of energies[7] of topical interest to modern society. In fact physics, hard and inaccessible as it may appear with its many forbidding formulas, nevertheless can be anchored in the imagination of the general public through the latters familiarity with many of its sophisticated phenomena. A familiarity which often is totally false (how many people in general have even an approximate idea of what an atom is, yet everyone nods in full agreement whenever atoms are mentioned). In mathematics on the other hand, most of the basic concepts, apart from numbers and triangles and such things, are totally unknown, and when it comes to more sophisticated concepts, there is a total ignorance in striking contrast to the natural sciences.

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[6] The KdV equation became very fashionable at the end of the seventies, when unexpected connections to totally different parts of mathematics were discovered. It does not play a crucial role in the mainstream of theoretical physics, but it does point out the treasures to be found on the sides. Mathematicians are far more open to such diversions than theoretical physicists who on the oether hand are far more focused in their activities

[7] If energy is indestructibe, how can we then ever talk about an energy crises?