The Fractalist

Memoir of a Scientific Maverick

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Mandelbrot is full of himself. Among mathematicians it is hard to find anyone to rival his unabashed egocentricity. But so he is a maverick. Many find him downright obnoxious in his self-promotion, others find him charming, maybe because he expresses openly what many only think in private. Whether egocentricity is a virtue, is a matter of debate, however any autobiography is the result of an egocentric impulse, and as such not only forgivable, but essential. You read a memoir, not so much for finding out cut and dry factual truth, but to, if momentarily, inhabit the skin of another, and savor that special perspective only an individual can provide.

Mandelbrot is very much hung up on success and recognition, in short on fame. His overriding ambition is to become a scientist, to make discoveries that matter, to become a household name, in short to become one of those legendary white males of the past everyone believes dead[1]. Any memoir invariably engages in name-dropping, and Mandelbrot's is no exception. To be socially acceptable, it seems as if one needs to be a Nobel prize laureate (or its equivalent) or at least to be closely related to one. But let us begin from the beginning. Mandelbrot often bragged that his career had not been straight, but followed a fractal curve. The memoirs should provide an opportunity to more closely follow the nature of his special fractal destiny with its ups and downs and digressions, what could be more boring than a straight line from the cradle to the grave? In particular we expect it to vindicate his claim.

Central to his tale is a remarkable photograph showing seated in the bosom of his family three distinguished French mathematicians - Hadamard, Montel and Denjoy. It was taken in 1930, when Mandelbrot was just growing out of his colts, and set in Warsaw, and the occasion a stop over en route to a conference, in fact the First Conference of Mathematics of the Union of Soviet Socialists Republics, to be held in Kharkov. The central figure of the picture, in fact constituting the intersection of his family with the larger scientific world, is the host - his uncle Szolem, who was to play the part of both father-figure and older brother (being a much younger brother of his father) but above all his scientific mentor, against whom he was fated to rebel. All in all one can see the roles of the three French mathematicians as that of fairies present at his inception.

Mandelbrot is of Jewish stock with roots in the vaguely defined regions of Polish and Lithuanian lands under Russian sovereignty during the 19th century. This setting constituted a confusing ethnic mosaic with a messy history few Westerners care to concern themselves with. As an illustration of the confusion one may remark the tragic-comic

[1] The last time I met him, he proudly referred to having been thought long since dead, along with the likes of Newton and Leibniz

fact that Polish speaking Benoit could not communicate with his paternal grandfather who spoke Yiddish. The Jewish presence was marked, and their traditional ambition of reaching spiritual and intellectual prominence through preparatory commercial success has been amply documented in the literature[2]. Those elements of the Jewish destiny are very pronounced in the Mandelbrot saga. His father was indeed toiling as a clothes merchants with many a setback which would have broken a lesser man, while his younger brother, the aforementioned uncle Szolem, was left free to pursue his intellectual interests. Mandelbrot's mother was an independent woman for her times, choosing the practical pursuit of dentistry. One surmises that the overriding ambition of the young Benoit and his brother was to do very well in school, something that might have been so obvious that the author does not even dwell on it. In fact Mandelbrot does not focus very much on his Jewish background, perhaps it is indicative of an egocentric personality to play down his collective identity. Of his childhood and he has not very much to say, although in the personal reminiscences of an elderly person they play a very dominating role. He is content with presenting some evocative scenes from a vanished world of the 30's, in particular a sojourn on a farm in present day Belorussia must have been a heady experience for a city-boy.

At the age of twelve his family settled in Paris. This was surely a momentous change and necessitated learning French which he did with aplomb, making it almost his native language. They lived in very modest circumstances, quite a contrast to life back in Warsaw, and although they had already had family settled in the city, among them the ubiquitous Szolem, the author confesses that with this move he for ever lost the sense of physically belonging to his surroundings, conducive no doubt to his prevailing sense of being a scientific outsider from which he has so much profited. The sojourn in Paris was cut short, the onslaught of the Second World War and the ensuing German occupation forced the family to seek refuge in the austere backwoods of France, namely the small town of Tulle. As noted the author does not play up his Jewishness, and treats the war years in a rather undramatic and factual way, although his and his family's position was of course very precarious. But by keeping a low profile, using forged identities and relaying on the solidarity of the French public, they all survived unscathed, even when the situation tightened at the end of the war and the Germans also took over the ostensibly independent Vichy administration. In fact he managed to continue his education in Lyon, only temporarily being diverted to manual work in instrument shops or out in the country side. Experiences he turned to his advantage. Throughout this time he shows the characteristics of a star student, with very useful cognitive skills such as remarkable powers of memorization.

The French school system is very elitist. People are ranked according to scholarly performance and the pinnacle of success is entrance to one of the elite schools through a rigorous examination process for which usually a year or two of intensive cramming is required. Once you are through you are set for life. You belong to the aristocratic educational elite a distinction that like a birth-right can never be taken away from you. It is indicative of the social changes of the past sixty years that many of those graduates were content to be high-school teachers, which nowadays are looked down upon. One may have strong opinions on the wisdom and merit of such a process, but it is an inescapable

[2] Stefan Zweig in his memoirs 'Die Welt von Gestern' eloquently explains it

fact that most French mathematicians, with some notable exceptions, were beneficiaries of that system. To Mandelbrot it fitted beautifully and he refers to himself as the academic wonder of his year. In fact, he claims, he performed better than was what considered physically feasible. His explains it all by his superior geometric intuition, enabling him to see through senseless obfuscations in the formulation of the problems and get straight to the heart of the matter. And so he was accepted at Ecole Normale Superieure and his future as a mathematician was cut out for him, along the lines of that of his uncle. And it is now that he makes the most important decision of his life. After a few days he decides to drop out of ENS and instead enlist in the Ecole Polytechnique, much to the consternation of his uncle. The reason for this, surely amply rationalized retrospectively, was a rejection of classical French mathematics as epitomized by Bourbaki and the figure of Andre Weil. Mandelbrot makes a distinction between people of the ear and of the eye. He belongs to the latter as testified by his obsession with visual geometry. While algebraists and other formal mathematicians in his view, are dominated by the ear. Characteristic of the latter is their ability, like that of Mozart, to reproduce on their first try perfectly, while the eye people stumble and bumble along and only gradually arrive at what they are desperately aiming at.

Rejecting Bourbaki Mandelbrot became an applied mathematician in spirit. He did well of course at Polytechnique and he chose to continue his studies at Cal Tech and Pasadena engaging himself in aeronautics. In retrospect this might seem to be an eccentric diversion when put on a CV, but he makes it seem very natural. The immediate postwar years were an exciting time with an unbridled optimism as to the blessings of technology including that of nuclear power and space exploration. The former has been almost universally castigated while the latter has become rather marginal, but in my childhood of the 50's rockets and going to the moon were truly exciting things indeed. The term 'rocket science' as being the ultimate, is surely a linguistic fossil stemming from that period. Flying was obviously connected with fundamental practical problems of a sophisticated mathematical nature, turbulence being one enduring example, and one which has intrigued him throughout his career. What was a more natural path to follow after his years at Polytechnique? Then there was the seduction of sunny southern California, which would in succeeding decades allure massive immigration, and a hotbed of a new generation of emerging scientists. He notes in particular the Prussian physicist Delbrück, who more or less singlehandedly created an entire new discipline - physical biology, or biochemistry as it would later be called, something that must have served as a pivotal inspiration to the ambitious young man. This was followed by military training and then somewhat belated graduate studies at the university of Paris. Nowadays a doctorate is a necessary prerequisite in any academically flavored career, not at that time. His uncle urged him to settle down and produce something, it was a shame that at his age and with his talents not to have written a thesis. But what to write upon? The perennial problem of almost every graduate student[3]. By chance his uncle, somewhat contemptuously, handed him a

[3] In contrast to most sciences this is also somewhat of a problem for the advisors as well. Some advisors elegantly solves it by arguing that if the student cannot come up with a problem on his own he is not cut out for his task.

paper, asking him whether he could make any sense of it[4]. It turned out to be a paper of a linguist Zipf, who had noted a peculiar regularity in the vocabularies of texts, namely that the product of the rank of a word (in a list of decreasing frequencies) with its frequency is a constant. What was remarkable was that this was independent of language and the nature of the text as well as the competence of its author. Could this be a law in the social sciences that has the same kind of universality as one hitherto only have found in the physical sciences? This greatly excited him. Zipf did not get everything right though he remarks, and his law had to be modified to the reigning Zipf-Mandelbrot law, but what those changes actually amounted to he does not disclose, which is somewhat galling, but of course in compliance with the ambition of writing for a general public. This became the basis for his dissertation, which was hastily put together and hardly a big success, yet, as he puts with characteristic modesty, way ahead of its times. After his belated dissertation he was ready for the classical grand tour, or as it is now referred to as a Post Doctoral. It took him once again across the ocean, but this time to MIT and the Institute at Princeton. The year is 1953 and Mandelbrot is almost thirty. At MIT he encountered the same kind of excitement he had found earlier at Cal Tech. Wiener and Shannon were the inspiring figures, cybernetics and communication were hot topics, and the technological no-nonsense attitude had also invaded traditional humanistic disciplines such as linguistic, through the figure of Chomsky. The early fifties may very well have been the pinnacle of the MIT era, at least if one trusts Mandelbrot. At the IAS he had the distinction of being von Neumann's last Post Doc. Another towering figure was Oppenheimer, who hard pressed to arrange talks for the ladies and faculty of the humanities, had while on a common train ride suggested to him to lecture. The lecture was a disaster and a triumph. The mathematical historian Neubauer had attacked his lecture viciously only to be rebuffed by Oppenheimer and von Neumann in turns. Mandelbrot retells the story with great relish maybe not fully appreciating the condescension involved.

Thereafter Mandelbrot returned to Paris on a CNRS stipend, met Levy, and then married and started a family. Ambulatory years followed. There was a stint at Geneva working, if perfunctorily with Piaget, the famed pedagogical theorist, as well as meeting Kac and Feller. By the mid fifties the postwar expansion of higher education was under its way, and with it a growing demand for university teachers. Anyone with a Ph.D. was fair game, and Mandelbrot landed a position in Lille, while living in Paris, and thus of course being the first turbo-prof (setting yet another trend?). And so in 1958 he went to IBM ostensibly for a summer job and ended up staying there for thirty-five years until he was offered a special professorship at Yale. This suited him well, and in fact the bulk of his scientific work was accomplished during this period, so in fact IBM far more than any other institution is connected with his name, something that no doubt was a source of embarrassment to him. When he lectured at College de France in the early seventies, his affiliation was considered unsuitable and an alternative one had to be arranged. IBM was no doubt doing well during the time computers were bulky and not yet in the possession of everyone and they could afford the luxury of having people just hanging around thinking of things that excited them with no pressures of formal publishing. Lecturing and

[4] One can almost see the uncle dropping the crumpled paper with an expression of total disgust, with his nephew scrambling down on the floor to pick up that precious crumble from the rich mans hand.

teaching were encouraged and IBM gladly patched up the salary decrease that a university visiting appointment would normally entail. There were occasional visits but only at top institutions, and one surmises that he was impatiently waiting for a call from Harvard or its equivalent. A position at Chicago fell through as he unwisely, not to say naively, as he explains, revealed that he was not just king of one trade. When he finally was embraced by the academic establishment he was almost seventy.

The latter part of the life of a successful career is often boring, at least to read about (but as Simone Weil reminds us, happiness is simply wonderful to experience non-vicariously). There is no drama left, no struggle, only accolades and honors bestowed. And Mandelbrot loves it. Recognition finally came his way, his success is complete, he is dined, prized and revered. People even think he is long since dead, as noted above, surely the ultimate in accolade. But what is really his scientific accomplishment? Mandelbrot himself with usual modesty refers to Keplerian insights, referring to Kepler's final abolishment of epicycles in favor of at the time obscure ellipsis. How many did he have really? Quite a few. Definitely more than Kepler.

Pure mathematicians tend to dismiss Mandelbrot as a mere salesman. The mathematical concepts he pushed did not originate with him but had a very long pedigree, The Swede Helge von Koch introduced the snow-flake before Mandelbrot was born. And Cantor and his eponymous set surely was if anything of a fractal character, in fact one of the most basic and elementary examples of self-similiarity. Likewise the concept of Hausdorff dimension was established long before Mandelbrot engaged himself in fractals. However, such dismissal is a bit snobbish and smacks of sour grapes, and most importantly misses the point. Mandelbrot never denies his predecessors, nor makes any inflated claims of originality as to the mathematics. On the contrary he takes pride in discovering early precedents not only in mathematics but also in the arts, seeing those as confirmations of his vision. The presence in art most be particularly gratifying. A drawing is often more revealing than a photograph, although the latter is seen as objective and exact. The draughtsman has in some sense to really understand what he is seeing, not mindlessly copying it. He needs to discern basic underlying structures and depict them clearly. Therein lies the scientific value of say the drawings of a Dürer or a da Vinci. Thus it is hardly surprising, at least in retrospect, that artists would have discovered the fractal nature of trees and other objects in nature. Now Mandelbrot claims that even if the objects he brings forth have been known to pure mathematicians, they tended to be disgusted by them as mere pathological monsters, and it is he who showed how natural and useful they really are for the study of nature. Rather than those being exceptions, the classical mathematical concepts such as straight lines and circles are the real exceptions. Where do you find a straight line in nature? The horizon dividing an expanse of water from the sky? Were do you see a circle? The sun, the full moon or the iris of an eye. Does this complete the list? However, it is true that when explicit examples of functions with no derivatives were exhibits many mathematicians recoiled in horror. Yet they quickly understood that those were the norm and the whole theory of Lebesgue integration was invented to deal with them. In fact much of local hard analysis deals with very pathological functions. On the other hand well behaved classical functions such as elliptic and theta-functions are nowadays interesting mainly to the algebraists. The real contribution of Mandelbrot was to have a unifying

vision based on a very good eye. He describes himself as a visual geometer, who during a long life has collected a vast mental filing cabinet of visual images, through which he has been constantly rummaging. This may go some way to explain the rather remarkable intuition he has shown in guessing the dimensions of certain sets. Unable himself to come up with technical and rigorous proofs he has nevertheless been vindicated, and the efforts to do so have turned out to be substantial.

Mandelbrot is by temperament an applied mathematician, and his zoo of fractals are primarily offered as examples of modeling. It hardly needs to be remarked that fractals themselves are mathematical idealizations which have no literal counterpart in the natural word, due to the cut-off on the atomic level. In the context of applications one should not fail to remark that some of his insights have gone beyond fractal images. One example is the modeling of commodity prices, which he claimed did not follow the standard Brownian motion with Gaussian distributions, but hyperbolic ones and not independently but showing some traces of memory. In particular this led to models with much bigger fluctuations, more in accordance with observations.

Characteristically he chose not to publish himself through technical articles but instead through a coffee-table book - The fractal Geometry of Nature, which, in the words of its author, is in the nature of an extended essay. It was published shortly before the demise of his uncle Szolem who wondered what exactly the intended audience would be. Mandelbrot replied that he had no idea, but expected it to create itself, and that it certainly would be very different from the fifteen experts or so, that his uncle wrote for. Not knowing his audience, has forced him to write clearly and understandably, a task that he confesses has not come easily to him. Whether true or not, his popular writing with their striking state of the art computer generated pictures, did make him famous among the general public, a rare feat for a mathematicians to achieve

The ultimate fractal object is of course the Mandelbrot set. It started out already at the turn of the last century through work by Julia and Fatou studying the iterations of the quadratic map $z \mapsto z^2 + c$, further work was done in the 50's by Brolin, a student of Carleson. Those pioneering works were hampered by the lack of cheap and powerful computing power. For each complex value c we get an interesting fractal set, the so called Julia set of c. However, the real interesting thing happens when we consider the entire parameter space - the complex plane. And the Mandelbrot set is defined as those c which correspond to connected Julia sets, or equivalently for those c for which the orbit of 0 remains bounded. Mandelbrot choses not to give a definition of his set, for fear of antagonizing readers, but remains content by referring to the quadratic polynomial. The Mandelbrot set, which is clearly an object of pure mathematics, has been described as the most complicated mathematical object known to man, and by many philosophically minded mathematicians proposed as a vindication of the Platonic view of mathematics. An object discovered and not invented, its intricate structure being a complete surprise. Its existence, as the author muses, throws into doubt the classical definition of complexity as introduced by Kolmogorov and elaborated on by Mandelbaums college Chaitin at IBM. How could such complexity arose from such a simple algorithm? One may remark that the same holds for the prime numbers, which are also selected by a simple rule. However, the complexity of the primes has so far not been visually manifested, and there is nothing as the eye to

instinctively appreciate structural complexity. Much of the immediate seductive appeal of the set is also due to the various color schemes, which have in general no mathematical significance.

Mandelbrot was not able to see his memoirs into print and hold his ultimate book in his hand. A foreword has been supplied by his widow, and an afterword by Michael Frame.

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