Funktion, Begriff, Bedeutung

Fünf logische Studien

G.Frege

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Frege is hard to get hold of in the original German. His collected works have been published in an English translation. He is both a philosopher and a mathematician, with the predictable consequence that he tends to be shunned by both. To a mathematician his analysis of general concepts may appear too metaphysical, while to a philosopher his fondness for formulas may appear too technical. Natural language is not logical, it thrives on ambiguity and self-reference, making the line between the profound and the misleading razor thin. This is an unacceptable state of affairs. The aim of Frege is to create a more formal language, in which such pitfalls are entirely avoided. This calls both for an analysis of natural language (by most people represented by your mother tongue) and a creation of a symbolism. The former may appeal to the philosophers, the latter may appall them. (With the mathematicians it might be the other way around.)

The five essays are very brief, and they are introduced by a lengthy foreword, in which the editor - Dr. Dr. h.c. G.Patzig (prof. em at Gttingen) among other things propagates the sympathetic opinion that mathematics is really part of the humanities, something which has been obscured by its potent applications to Natural Science. They are brief and lucid, but also rather condensed especially at the end, as if the author has started out leisurely, laying out the principles, dwelling on the obvious, only to find himself as the typical lecturer, cramped for space, at the end, in which he really have to cram in most of what he wants to say. Thus prose like this requires careful study, to be read slowly, and preferably many times. Do the rewards of such a reading really counterbalance the effort? What do we learn that we did not know before, and if so is it worth knowing or is it just pedantry? In general we do not learn passively, we learn by asking questions and comparing the responses to those we expect. Any philosophical text has to be read in a context of questions asked and ambitions outlined. Sometimes the text itself incorporate those, most often it does not.

In the first essay - Funktion and Begriff, Frege presents a clear notion of the mathematical concept of a function (redundant information to the modern mathematician who has digested it by his or her mothers milk) and suggests that concepts should be thought of in the same way. To Frege the notion of a proper name is made much wider than we ordinarily allow. To him a sentence asserting something is a proper name, either for truth, if it is correct, and for falsity otherwise. In this way ordinary formulas involving an equality sign will still be able to be considered as functions, the difference is now that they only take two values true or false. (Thus both x^2 and $x^2 = 1$ are well formed as functions, the former taking numerical values the latter only speaks true or false.).

This leads to the main essay, that of 'Sinn' and Bedeutung'. A given entity (*Gegenstand*) can have many expressions, they differ not in meaning ('Bedeutung') (the thing

to which they point or equivalently being the proper name of) but in sense ('Sinn'). A typical example given by Freger being the morning and the evening star. They have different senses but points to the same object, namely the planet Venus. (The assertion 'the morning star=the evening star' is not a tautological statement, unlike 'Venus=Venus'). The 'Bedeutung' of a sentence does not change, if elements of it are replaced by bother elements of the same 'Bedeutung', although if they have different 'Sinn' they do of course change the 'Sinn'. Now one has to be careful. The statement 'X believes Y' will have a meaning as soon as for X and Y entities with meaning have been substituted. (Hence if 'X' is the 'the presnet King of France', and 'Y' is 'Ceasar is bald', X refers to no entity and hence has no meaning, the same will hold of the complete sentence. This should be contrasted with the more modern empty-set interpretation, which would be that if you are a King of France you will believe that 'Ceasar is bald'. This is true, because otherwise there would be some King of France who did not believe it, and as there are none, that could npt be the case.) However the truth, i.e. the meaning of the statement will not be invariant of the meaning of the various Y. X may believe some true statements but disbelieve others. In indirect position the meaning of a sentence changes from that of being true or false to becoming more exactly the 'Sinn' of the sentence. This is a very elegant idea.

The three concluding essays are of more marginal interest some of them being in the nature of rebuttals to critics and thus involving the usual clarification of terminology and how it differs from that assumend and accepted by the dissenters. As noted before, the idea of a function does not have to be explained to a modern mathematician. However Frege points out to some inconsistency of terminology. We should really make a difference between $f(x)$ and $f(\xi)$ say. In the former case x is a variable (Frege abhors this terminology incidentally and prefers argument) for which different things can be substituted, in the second case it has just a 'dummy' function. This is exemplified by the awkwardness of the expression $\overline{dx/2dx_{x=2}}$ as we cannot substitute for x the value 2 without making the expression senseless. This is awkwardness indeed, but for a mathematician nothing else. For the mathematician, as the instinctive user of a natural language, intentions take precedence over expressions, and as long as they know of what they are talking and what they want, inconsistencies in form are rather incidental. The urgency of the creation of formal languages rests on not fully appreciating this.

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