Mathematics

The New Golden Age

K.Devlin

`December 17-20, 2009`

The New Golden Age for mathematics, or is mathematics the new golden age for us all. The title is ambiguous, whether intentionally or not, I cannot say.

Of all the sciences, to the extent mathematics is a science, mathematics is probably both the one with which the public has the most intimate connection with, and also the one of which it is the most ignorant. This makes the popularization of mathematics into a special kind of challenge. While most people have some idea of modern physics, or at least that it exists with momentous consequences for mankind, and similarly are versant with many concepts of modern biology, such as cells and genes, the notion that there exists modern mathematics, and that it is as diversified, if not more than any other scientific subject, to the extent that to even a professional mathematician most other areas are opaque, is not generally appreciated. How does one get this across? Martin Gardner tried heroically for many years running a column of mathematical games in Scientific American. His starting point was that people have in fact some intimacy with elementary mathematics, and started from there, exploiting the fact, that while in other subjects the reader has to take things on trust, not so in mathematics. The arguments are transparent, at least in principle, and accessible to anyone, at least up to a point. But of course this is enough to provoke a genuine curiosity, just as it is for the professional mathematician listening to a lecture. Later efforts have been made by a few select popularizing mathematicians such as Ian Stewart and the author of the present book, writers who have caught the attention of publisher and public alike (the connection is of course commercially intimate). So the task Devlin has set himself is to present to the general public (maybe in practice scientists in other fields, as well as mathematicians themselves?) some new developments in mathematics of the past century. The choice tends to be canonical, although as noted above, modern mathematics is very diversified.

Prime numbers are of course simple to introduce, and very quickly intriguing facts can be divulged, including examples of very big primes. (The proof that there is an infinitude of primes is of course so simple and elegant that no professional mathematician can afford to pass it up.) This topic, seemingly of pure mathematics, has had in recent years unanticipated applications in the business of securely encoded transmissions over the internet, which certainly adds to its topicality. Any serious popularizer also feels the obligation to bring in the Riemann hypothesis, and Devlin is no exception, but he wisely postpones that connection to a later chapter. The difficulty of factorizing numbers, as opposed to multiplying them, leads to algorithms, running times and P and NP, another subject that is, or ought to be very accessible to the untutored public, addressing some very basic questions with some fundamental ideas needing little if any technical preparations to be fathomed, the exploration of which invariably belongs to the most satisfying chapters on popularized mathematics.

Another commonly pursued thread of introduction is topology, in the case of this book also including graph theory. A large section is devoted to the four-coloring problem, maybe the most commonly appreciated long-standing conjecture and definitely the least interesting of them all. In fact the author decides to treat it with such thoroughness as to actually include a proof that five colors suffice. The amazing thing about it, is that the corresponding problem for higher genera surfaces was so much easier to settle. One may be a bit skeptical as to how much the general reader might have the energy to follow the details, but on the other hand there seems to be a vibrant subculture of amateur mathematicians versant in such disciplines, and who no doubt would benefit from it. No matter what the Appel and Haken achievement highlights a controversial aspect of mathematics, namely the limits of argumental complexity. From a purely Platonistic point of view a long chain of arguments, too long for a human to digest and check, is of course fully legitimate. Platonist perspective of mathematics being completely inhuman. But it raises the uncomfortable query whether proof simply is just a higher form of computation. As human beings we appreciate mathematics for the beautiful ideas which we can fathom, not for their mechanical implementations. This ties in with other topics the author treats, and which we have already mentioned, but he chooses (perhaps wisely) not to pursue them too deeply.

Topology offers also other gems, such as knot theory and surface classification, and the unknotting of the embedded genus two curve would challenge even a professional mathematician not accustomed to think on such matters. Group theory is another must, and with the spectacular achievement of the recent classification theorem, surely something that deserves prime of place. But as usual the presentation becomes a bit breathless, ideally the uninitiated reader should be allowed several stages at which to digest the material before being urged to consider the next. This is what education is all about, the slow and steady accumulation of a large set of data intricately structured. Thus one suspects that those introductory chapters works best on those who have already had some exposure and can benefit from a retroactive overview.

And finally having already touched on Riemann and the Four-color problem you can hardly ignore Fermat, especially as the book was written in 1998 in the aftermath of Wiles breakthrough a few years earlier. Fermat's conjecture is of course a curiosity, made important not so much by its intrinsic interest as by the legend and the social effects of the individual efforts involved in its solution. In fact the remarkable thing about it, is that in spite of its frivolous nature, it has generated so much genuine mathematical inquiry, to a large extent driving the development of algebraic number theory in the middle of the 19th century. The fact that it had such a simple and unexpected tie to the modern theory of elliptic curves almost makes you believe not only in the Platonic nature of mathematics but also its providential nature looking out for its disciples[1]. The presentation of the prehistory

[1] Wiles is a case in point. Dreaming about solving the conjecture as a boy, a beautiful instance of timidity wedded to audacious ambition, he was as a student led into mainstream number theory, only to eventually discover that the expertise he had so fortuitously acquired, was in fact beautifully designed to make him fulfill his old dream. No wonder he could retreat into intense seclusion, serving his Rachel for seven long years

of the final solution is masterly done, but maybe invariably much more beneficial to the fellow mathematician than the proverbial man in the street.

Finally there is fractal and chaos. The author making a rather detailed study of the Koch snowflake (although for some obscure reason not referring to the shape as that of a snowflake) sets the tone. There is even an ambitious attempt to explain what the Mandelbroit brain is all about and a nice motivation starting from a simple recursive problem to motivate the study of the iteration $z \mapsto z^2 + c$. This is again nice and intriguing mathematics starting from almost nowhere. But as such maybe more appreciated by the fellow mathematician, who can take heart that there is still tracts of wilderness in mathematics, innocent of elaborate mathematical technology[2].

`December 21, 2009` **Ulf Persson:** *Prof.em, Chalmers U.of Tech., Göteborg Sweden* ulfp@chalmers.se

[2] At least this was my initial reaction when I heard Douday lecturing on it at a Colloquium at Columbia in the late 70's