Goodbye Descartes

The End of Logic and the Search for a new Cosmology of the Mind

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Materialism is about understanding the world bottom-up, Idealism to consider it top-down. The former, although traditionally something most people feel uncomfortable with, is the basis for all scientific and technological advancement of humankind. The latter is a phantastic theory, too phantastic for most people to swallow, although Berkeley made quite a valiant attempt in his Three Dialogues to convince the reader that materialism was quite as phantastic when carried to its extreme, and providing an even greater insult to mans rational understanding. Ideally we would like to have it both ways. One such solution is the Dualism of Descartes, namely to posit two worlds, one mental of the mind, and one of matter, separate yet in some sense parallel. Because Descartes is a hard scientist, devoted to precision and intellectual honesty, his scheme is up for attack. How exactly can one understand the relation between mind and matter? Heidegger, on the other hand, also takes exception to this division between an objective material world and a subjective ideal, claiming that either point of view is too limited, that we need to take both. This is sometimes referred to as the holistic point of view. As Heidegger is a far more woolier thinker than Descartes, less devoted to precision in thought (not to mention moral integrity), his cop-out of having the best of two worlds is seen as profound. Maybe because this is the view most of us instinctively embrace[1]. What is but vaguely seen cannot harbor glaring contradictions.

The story starts with the Ancient Greek, to those we owe not only daring metaphysics, devoted to discover what lies behind the multifarious world of the senses, the most poetic expression of such an ambition being the well-known speleological simile of Plato; but also the axiomatic method of inquiry and discovery. What is so great about this method is its dialectical aspect, namely that by reason alone to unravel the mysteries of the world. Its most successful and beautiful manifestation is mathematics, and it is this allure of the power of the mind, that no doubts explains the great attraction mathematics has to people, who can fully appreciate the allure. In a sense materialism is a celebration of the mind, the mastery of mind over matter, making the latter completely transparent, at least in principle. However, there are problems when the mind turns to itself, can the mind fully understand itself? Is this not an impossible self-reference leading to logical paradox and ultimately collapse?

Aristotle tried to systematically study if not the whole mind, at least its reasoning power, being in a sense its most noble aspect, and the key to which we acquire truth. The result was his classification of syllogisms, in which he incidentally made some slips which went undetected for two thousand years, maybe because no one really cared that

[1] The present author of this review not excepted.

much. Other attempts were made by the Stoics, and as they lacked proper notation, their insights if correct were clumsily presented, finding themselves in a similar situation to the Babylonians who lacked algebraic notation to succinctly present their algebraic discoveries. Yet for most of western history, the study of logic was disdained, being seen as a stratagem of persuasion and seductive rhetoric, more suitable to the court than for the disinterested pursuit of truth. Especially Plato was suspicious of such skills, and Socrates talked with disdain of those so called sophists, who were so indifferent to truth, that they were as happy arguing one side of the case as the other, as long as they could dazzle with their brilliance. C.S.Peirce characterized mathematics as a study of necessary truth, not one of the logic of that necessity, claiming that a mathematician does not need to know the rules of logic, he applies them instinctively anyway. And besides, coming to think of it, is there nothing fishy about studying logic by logic, assuming what you want to know, a vicious circle, a case of self-reference? One legacy of Greek logic is the paradox of the liar, something which would haunt mathematicians until modern times (and beyond?). Variations of this self-referential paradox has been used to obtain positive results. St Augustine proved the existence of absolute truth, by making the statement to the effect that no absolute truth existed and getting a contradiction by assuming that this statement was true. It is not clear what has really been achieved, and the proof has a very scholastic fervor reminiscent of the ontological proof of the existence of God; yet it was collaborated upon by Bolzano in the early 19th century, and is often inveighed against modern day sophists, i.e. post-modernists.

A great exception to this was Leibniz whose ambition it was to create a formal language free of the ambiguities and imperfections of ordinary human ones, and which would allow a calculation, so whenever there was a difference of opinion one could simply sit down and make the necessary calculations to see who was right. Something like this is of course the case with mathematics, when the resolution of a controversy is made to the satisfaction of both parties, even the one being in the wrong. Because unlike other testy differences of opinions, the very objectivity of the resolution prevents it from being taken personally. In fact by the very process of seeing his mistakes the mistaken party assimilates the truth and makes it his own. Leibniz started out by introducing algebraic notation and proving some more or less self-evident truths, but he did not surprisingly get very far. Leibniz ambition was to some extent realized a century and a half later by Boole. The point of a formalization is to manipulate it, and Boolean algebra became in a sense the foundation for the modern digital computer, in fact mechanical implementations were made already in the same century, but they were ahead of their time, the right kind of technology to produce the appropriate hardware not being available until the end of the Second World War. But even if the hardware did not yet exist, the notion of software started to form[2]. The 19th century was a century of emerging rigor in which notions like limits and real numbers achieved their present foundations. This also led to a search for a firm foundation of mathematics and a renewed interest in logic, especially after the emergence of non-Euclidean geometry. Peano suggested axiomatics for the natural numbers, driven by the induction axiom, which allowed strict deductions of the basic laws of arithmetic,

[2] Of course the computer has a long pedigree, mechanical looms programmed by in essence were connected strips of punch cards had been around for a some time.

and more to the point Frege sought to reduce mathematics to logic, in particular giving a firm foundation for the integers. This should be contrasted to the opinion of Peirce, who thought that the integers were a more basic concept than logic itself, and he would in a sense be proven right. The efforts of Frege were later brought to some fruition by Russell and Whitehead, producing what must be one of the great dead-alleys of mathematics. The actual reduction of mathematics to logic proved to be a monstrosity which mathematicians tended to disregard totally[3]. But the ambition of formalization was nevertheless very much alive, and Hilbert is often seen as its most distinguished proponent. He is famous for his observations that objects of an axiomatized theory has no meaning whatsoever, what is important is only the logical structure imposed by the axioms. This takes all sensuality out of mathematics, if lines and points can be replaced by tables and beer mugs does it not turn mathematics into a frivolous game with no ultimate meaning?Hilbert was, however, no formalist at heart, his attitude to mathematics was definitely a sensuous one in which the character of the mathematical objects very much mattered. His formalization was merely a stratagem of being able to realize Leibniz ambition at least for mathematics. Ultimately to prove its consistency and its completeness, meaning that all we were desirous to know we would be able to find out sooner or later by logical reasoning - *Wir müssen wissen, wir werden wissen*. After all even if mathematics is ultimately formal, formal systems themselves would be concrete objects to study.

And indeed, formal systems can be encoded into numbers and hence number theory, a rich sensuous mathematical object and study filled with meaning (thus making in a sense logic a part of numbertheory, which ought to have delighted a Peirce). Gdel did just that and exploiting the Cantor diagonal trick (the trick of almost self-reference, and maybe the most fertile idea in the whole of logic) to the ultimate, punctured all such hopes. This did not stop the pursuit of formalization, but it did put a stop not only to Hilberts program but also seriously hampered the metaphysical ambitions of the logical positivists, those philosophers whose overriding ambition was to rid the world of metaphysics altogether. Formalization would find an outlet in the exploding development of the digital computer, the prevailing vision being that it would be able to simulate thought processes so well, that according to the predictions of Turing (who also incidentally had shown the limitations of formalism in his solution to the Halting problem, a demonstration more accessible than that of Gdel) in a near future a human being would not be able to distinguish the simulated intelligence of a machine from the real one of a human. The project of Artificial Intelligence was born, whose success was initially seen as in principle inevitable. Now in practice initial promises were never made good upon, the difficulties which in the beginning had been easily overcome turned out to become intractable as soon as problems of any degree of reasonable complexity was tackled. More and more of the early proponents jumped the boat and by the last decade of the 20th century AI was more or less totally discredited, although of course fragments of far more modest ambitions survived, such as various expert systems

[3] The great French mathematician Poincaré, was very contemptuous of logic, regarding it as a totally sterile field, remarking sarcastically apropos the Russell paradox that at least it now produced contradictions, yet recognizing the paradox as merely a modern reformulations of the old Cretan Liar paradox. Once when told that with one false statement one could prove everything, he is reported to have remarked that this does not surprise anyone having had to supervise a doctoral thesis.

discussed at some length by Devlin. Suffices it to say that the problem of vision has turned out to be extremely difficult testifying to the subtlety of the human brain, especially when it comes to specialized tasks such as recognition of faces. Those developments have been gratifying to most of us, because after all we recoil from the notion of being reducible to something mindless. Of mind being made out of matter, as opposed to the spirit made flesh.

Yet can it be done in principle, even if it may never transpire in practice? The computer, or rather the algorithms that drive its softwares, are good at checking things, i.e. following deterministic rules, far worse at coming up with new things, although from the formal point of view, the latter is only a question of trying out a sufficient number of systematically generated possibilities. The velocity of light and the sizes of atoms provide limits to the calculational power of a computer, but of course would computers have indefinite memories and able to perform an infinite number of operations in finite time, it would outpower those of finite minds, although such intelligent simulations would have little to do with the workings of the human brain, except of course being conceived by the same. In some ways this grander picture, not the infinite one though, has actually been implemented. On one hand we have the mechanistic picture, explained already by Liebniz och Laplace, in which the positions of all the particles in the universe at one given time determine their positions into the indefinite future as well as the distant past. In short the world is deterministic. In a deterministic world we do not have the option of making perfect predictions, because those very predictions themselves may not be what is determined by the universe. In fact in a deterministic universe we may be fully convinced that it is undeterministic and chaotic, because we have been determined to believe so. Now as the paradigm has changed from a Newtonian world-view into one in which at smaller scales, determination of position and momentum cannot be arbitrarily accurate simultaneously, the spectre of such a state of affairs has to a great extent vanished. But there is another kind of determinism, the biological. What the Darwinian concept of a world evolving through natural selection really means is that this is given by an algorithm. This algorithm has produced intelligent life as our own and could in principle be capable of creating even more powerful intelligent life. It is an algorithm in principle, but of course it cannot be pinpointed and used mathematically[4]. It is a materialistic theory, and in which biological activity is to some extent reduced to biochemical activity related to the genes (but not only that, other non-genetic factors are important in the unfolding of life), but it is so difficult that the human mind may never in principle fathom it, thus making the problem of consciousness in principle reducible to the material world, but never in any sense that would makes sense to us.

After those flights of fancy let us return to something more down-to-earth, namely linguistics. Traditionally linguistic, with its roots going back to the late 18th century with the discovery of William Jones of the similarities of Sanskrit to European languages, was mostly a historical pursuit, whose object it was to trace the development of languages over time and in particular to discover its family tree and reconstruct extinct branches. The analogies with natural selection were striking, and as with most analogies ultimately

[4] Although it can be simplified and made precise in many ways and used to simulate on the computer all kinds of artificial life.

misleading. Chomsky was to change all that, bent on transforming linguistics from a soft science to a hard one using mathematics, no doubt encouraged by the success of formal languages and carried by a conviction that natural languages on some deeper level should not be that different. Devlin shows, at least initially, great enthusiasm for the contribution of Syntactic structures by Chomsky likening it to a great physical theory. What did he really do?

First there is the vision. Human infants are not born with language, but they learn to talk as naturally and as inevitably as they learn to walk. No formal instruction is necessary, and the speed with which they so competently acquire the sophisticated use and with such meagre input is astounding. The most natural explanation being that humans are prewired for language, just as they are prewired for their ability to walk. This is very suggestive and remains a very attractive idea. However, so far no neurological basis has been discovered for language, so far the only thing one has been able to do is to locate the language center geographically in the brain. In most cases this is located in the left hemisphere, supposedly the hemisphere for logic processing. Thus Chomskian linguistics cannot be done on the neurological level. What remains is to point at common structures of languages, the manifestations of which can vary so widely. This is done by positing the rules of a generative grammar, so named because it is supposed to generate all grammatically possible statements in a language (in practice an infinite one), to be supplemented by a transformational grammar that sees to the finer points pertaining to inflections and such things. By phrasing the rules of the generative grammar carefully, a universal grammar should be possible to articulate. Once again no neurological basis has ever been found for such theoretical constructs, and of course it is far from clear how such would be articulated on that level. The important point of view of Chomsky is to discard semantics, i.e. the meaning of sentences and utterances, and completely concentrating on syntax. This is a formalization of language, divesting it of meaning, and thus a purely materialistic conception of how to combine linguistic atoms. Every language speaks about nouns, verbs and adjectives, and those can be extended to phrases of the same. Thus some simple rules of how to combine noun-phrases with verb-phrases and how to extend phrases inductively are presented. To the non-mathematical student it may seem rather intimidating with abstract formulas, with arrows and trees, but the point of setting up a mathematical formalism is to manipulate it mathematically, otherwise it is but descriptive. What mathematical operations are done? None of what I can see, and if so, why should it be considered a mathematical theory? It might nevertheless have other uses. The first claim is that every grammatical sentence can be generated by the syntactic rules. How do you prove that? For that you need another characterization, be that one by rules as well, a proof is in principle possible; but if your linguistic intuition is the criterion for grammatical correctness (which in fact has been the guiding force behind fine-tuning of generative rules) there is no guarantee that someday some one will not produce an intuitively grammatical sentence which cannot be generated. One may of course also posit that by definition a sentence is grammatical iff and only if it can be generated. Thus generative grammar becomes a tool to analyze sentences and decide whether they are grammatical or not. If the process can be carried through to the irreducible components of the language, i.e. words, then it is defined to be successful, and the sentence grammatical.

One trivial objection can quickly be disposed of. The smallest irreducible components of a language are not necessarily words in the sense of being delimited by spaces. Words can be compound, especially by phonems, some of which are grammatical ornamentations, also a word can be disconnected, such as the French 'ne...pas'. The English determined 'the' has no direct counterpart in Scandinavian languages or Rumanian, but instead used as a suffix on the noun. (In Russian it is missing altogether in the literal sense, but of course determined noun phrases can be constructed anyway). Furthermore some idiomatic expressions, although consisting of several words, are in some sense to be conceived as single words. Now there is a big problem with this way of deconstructing sentences, in order to be able to do it, you actually need to know the meaning of the sentence. Furthermore, just as natural languages abound in words that can mean very many things, in English every noun can in principle be turned into a verb without changing the spelling, and conversely. (You can chair a meeting, and afterwards have a run), a phrase can be interpreted differently. Thus from a purely syntactic point of view you cannot analyze a sentence, the same sentence letter by letter, can be associated to different syntactic trees, and thus ultimately mean entirely different things. This might be annoying, but there is more to come. If this method of syntactic structure should mean anything, not just be an arbitrary one imposed on language, it must have useful applications because of it on some level reflecting something important about language. In particular if there is a common deep structure to all sentences, independent of language, this structure should be invariant under translation between languages, and thus in principle allowing translations, by breaking down a sentence into its constituencies, translating those and then applying the appropriate transformations peculiar to the grammatical surface structure of the target language. There might be some problem in the translation of individual constituencies, but everyone who has done some non-trivial translation knows that this is not the way it is being done, but before translations are possible, paraphrasings has to be made, the idiomatic idiosyncracies of different languages being so peculiar. How is 'Out of sight, out of mind' translated into various other languages? Maybe the whole thing should be just seen as an idiomatic expression, i.e. a single word. Or is the expression simply not grammatical, although often used. Thus is grammar not to important after all?

Attempts to automatize the syntactic structure of languages are constantly being frustrated. How to get out of this impasse? This constitutes the whole point of the present book. Formalization of natural languages, their reduction to formal syntactic structures on which various algorithms may be applied is doomed. With a certain amount of hyperbole, the author claims that this makes a tradition of two thousand years obsolete, and that we instead have to develop new tools and create a new science in order to understand human language and ultimately the workings of the mind. In particular that human language is not based upon the application of different rules, consciously or subconsciously? The basic thing in language is meaning. Only if we understand that language is based on meaning, the semantics as opposed to the syntax, will it start to make sense to us. The mistake of Chomsky was to concentrate on syntactics at first believing that just like in the natural sciences, we could at first simplify getting a basic approximation on which we could then build, when in fact, such simplifications give no real clue to what is going on and thus makes no progress. In retrospect one wonders why the author initially in his presentation

of Chomsky was so excited?

Now by concentrating on meaning we are in the position of the idealist who starts from the top. In what sense can a science be made out of such an approach, is this not the classical approach of literature and the arts? The materialist has the ambition to show that meaning (like consciousness) is an emergent feature of simpler phenomena. Are we to abandon all such ambitions of looking at the deeper features of phenomena? Devlin talks about soft mathematics, what is meant by that? He makes some vague references to thinking inspired by mathematics rather than being based on it. A new kind of mathematics provoked by mathematical minds when encountering phenomena which evades traditional approaches. In a way soft mathematics exists already, this is what mathematicians apply when they think of mathematics, coming up with mathematical ideas, as opposed to doing mathematics. This kind of activity is mostly subconscious and indeed driven by intuition and thus in any manifestable way illogical. But by being illogical and not given to precise articulation it is not exportable. Can it then be termed science, except by stretching the traditional meaning of science, as to make it lose all its intrinsic meaning. Science should be objective, i.e. on some level testable, otherwise it can be spread in all directions and become shapeless and thus meaningless. It is the constant pruning of scientific ideas which makes it penetrate deeply into the configuration space of ideas. There is essentially no indication of what this new soft mathematics actually would consist of, except a wishful dream. What you get instead is a description of what language communication really entails when meaning is paramount. The story you get is of course very plausible as well as speculative, like literature it provides a great explanatory structures, but is it true?

The story goes something like this. Language is about communication between individuals. Let us take the basic and most transparent case, that of the conversation. In a sense a conversation follows some unwritten rules, but if we would take the trouble to make them explicit, we would all find them eminently common-sensical. Language is about meaning, it refers to something beyond itself. This refers to the general context of the conversation, which could be split up into a most basic background that makes conversations as such possible, and a more specific and relevant foreground, which makes a particular conversation possible. Without this need of communication language would not have emerged. When we speak (unless we try out a learned language) we are not aware of any rules, it is the need to speak, and the meaning of what we want to convey, which makes us form comprehensible sentences, which incidentally are seldom grammatical. There is a structure to the written sentence which is missing in the spoken. It is not even clear that there is a clear distinction between individual words. A sentence can be interpreted in many ways, and the way it is interpreted is not apparent just by the sentence itself, but by the context in which it is embedded. Conversations are cooperative efforts, each participant making a contribution. If this is skewed the conversation will peter out. Listening is not a passive thing, on the contrary it is an active confirmation, and communication is not only effected by words but by many other things contributing to the ambient context. One striking thing about conversation is that, although at times there is an overlap in speaking, normally one knows when to stop, and knows when one is allowed to break in. There are obviously cues, some of them being obvious, but mostly I suspect subtle and

subconscious. Conversations are not just about the interchange of information, although of course a lot of information and crucial one at that can be exchanged, but also intend to serve other purposes, such as what is usually called 'small talk' to say nothing about lovers verbal carasses. A conversation sets up mutual expectations, if a question is asked one expects an honest answer, giving enough detail but not more than is appropriate. What is appropriate? Once again this belongs to the tacit contract. If a contract is broken it quickly leads to a breakdown of the exchange not seldom accompanied by recriminations. Sometimes the responses seem on the face of them absurd, this means that they should be interpreted ironically, or that the speaker is making the equivalent of a warning cry. In human communication most of what you here is expected, thus understanding is a matter of small interpretations. Sometimes someone literally says the opposite of what is meant and expected, but neither participant may notice it, or if they do, it is usually tacitly placed over, to bring it to attention would seem to be nit-picking. And one can go on and on like this. The most interesting point is probably the proposed fact that in a conversation two people share a large common reality through a shared context. Thus it also points to a supposed empathy between people, a high degree of unification. Language becomes the way to unite different minds, an extension of the interior monologue, except of course that the interior monologue may be an off-shoot of the emergence of language. This social cohesion among human beings, the need to play on the social scene, is not restricted to humans but probably shared by social animals such as wolves. You can communicate with a dog and a horse, not verbally of course, although animals may learn the appropriate responses to words and commands. This non-verbal communication could be quite subtle. Language allows it to become even more subtle.

Now what does this kind of thinking involve? Is it science? It is highly speculative of course, but that alone is not disqualifying, the crucial point is whether it is testable. Some of the statements are testable, others are articles of faith, which may have some explanatory value, but mostly fulfill the function of celebrating the subtlety of the human mind. Social interaction is a subtle thing, in fact most of the intellectual power is tied up with it (how can this be measured or otherwise corroborated?), and thus it goes to reason that if some of that awesome thinking power may be rerouted for other means, such as mathematics, the results must be impressive. (Or at least weird.)

It is clear that a context-free approach does not work for understanding the mind. The study of the mind is (as is quantum physics?) a self-referential activity, and thus fraught with traps and loaded with confusion. It is a good anti-dote to simplified approaches, but could it ever be tamed by some systematic inquiry, and what would be understood by understanding?

To return to the paradox of the Liar. How to resolve it? The classical solution is to prohibit self-reference as illegal, and in fact if you formalize a language it becomes syntactically impossible to impose self-reference on it[5]. Still normal human languages are self-referential, to prohibit self-reference seems to be throwing out the baby with the bathwater. But as Wittgenstein (this windbag) already remarked in Tractatus, the truth value of a sentence is not part of the sentence itself, to decide whether a sentence is True

[5] Some artificially designed languages, allow self-reference, in fact are designed with that purpose in mind, according to Hofstader's Gdel. Escher Bach.

or not, it needs to have meaning, thus to be seen in a context. Sentences do change contexts, which is obvious in the social sphere. Certain opinions you have of people you keep to yourself (or better still share with some trusted friend) knowing that if they would be publicly known they would change the context and often make themselves irrelevant[6] . The solution to the Liars paradox, according to Devlin, was not clearly stated until around 1986, in which it was concluded that the context in which it was uttered changed depending on the truth or falsehood of the statement. To my mind this is just a subtle way of evading self-reference after all by making a distinction between a sentence and the sentence set in a context, and thus maybe not quite so striking to me as to the author. But that is a minor quibble.

The notion of context or, as Devlin prefers, situation, is of course a crucial one, although not quite as recent as he makes it out. (I recall vividly this being pointed out by Ulam in his autobiography as a response to such kind of logical puzzles, or more generally the problem of minimal representations.). It is of course ubiqituous in post-modernism, the most social of all intellectual attitudes, in which the steady shifting of contexts, just as in social situations, makes indefinite evasion not only possible but almost the whole fun of the game.

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[6] A similar phenomenon is given by the Richards paradox. 'The smallest number not expressible with twelve words in the English language' is asked for. What is meant by that? How can you associate to an English expression a number? Ultimately by fiat, you simply are given a function on (a subset) of integers, this has nothing to do with the meaning of the sentences, although such meaning can of course inspire the setting up of the function. Once the function has been set up, the terrain has changed. With respect to this function there is a very definite meaning to the expression 'The smallest number...' which in a sense overrides the number formerly and hence formally made to correspond with it.