

# The Philosophy of Logical Atomism

*B. Russell*

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A guiding principle of Russell has been the epistemological problem of gaining secure knowledge. This led him into mathematics, where the prospects of such a quest seemed to be the most favorable. But even in mathematics he encountered doubts caused by sloppy thinking, especially in the haphazard way infinitesimal calculus was being taught. Arithmetic proved a better venue, and his ambition, partly inspired by Peano and especially Frege, was to derive mathematics from logic<sup>1</sup>, the culmination of which was his joint work with Whitehead - *Principia Mathematica*. This also provided the culmination of his intellectual work, and in his autobiography he complains of the total mental exhaustion those labors entailed.

Russell entered philosophy via mathematics. He was always to be a reluctant philosopher having no great esteem for philosophy, and especially not for fellow philosophers. The domain of philosophy, just as wilderness in North America was continually shrinking. Once a part of philosophy became clear and rigorous, it was open up for cultivation, meaning becoming part of science, and those minds who thrive less on exact and systematic toil, than on flights of speculative fancy, were forced to retreat to murkier remains.

Logical atomism is a dry and dusty subject, and Russell is very apologetic. Yet he makes his best in his lectures to present it in an attractive way illuminated by his wit and oblique insight. It is also clear that in his presentation he is very much influenced by the young Wittgenstein, the debt to whom he makes no effort to hide<sup>2</sup>. His point is that language influences thought, especially philosophical thought, and that so much that goes for truth and profundity in the latter, is actually an artifact of the confusing use of language. If one only succeeds in making language clearer, much of that confusion can be cleared up and resolved as so much phantom thought. However, by the very nature of the task, it is not an easy one; and what is worse not a very inspiring one, involving a lot of technical difficulty. In fact to really think about those things, especially about logic, is extremely taxing, as opposed to thinking of the associated symbols. Russell admits that during a six month period he is only able to think about logic, as opposed to its symbolic representation, for a total of thirty seconds. The real great minds, he speculates, maybe do it for a minute. Most philosophers though do not even think of it at all, because they are bad philosophers<sup>3</sup>.

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<sup>1</sup> possible once you had done it for arithmetics, as mathematics was being arithmetized systematically at the end of the 19th century.

<sup>2</sup> At the time (the winter of 1917-18) when he was giving the lectures he did not know whether his protoge was dead or alive. By the time of his afterword written many years later (in 1924) he must have read Wittgenstein's Tractatus which was published after the war, a book which must have confirmed his views. He then even more explicitly acknowledges his debt to the younger colleague.

<sup>3</sup> This reminds me of my theory using the Cantor set as a model for thinking. At any given time

Language is a problem, both its vocabulary and its syntax. The problem with its vocabulary is that there is no difference between words that design abstractions and those that design concrete things, misleading the mind to think that they are the same kind of words and have the same kind of relation to reality. Syntax is another problem setting up structural similarities between different propositions provoking spurious analogies. Russell suspects that this problem of syntax is particularly serious with Aryan languages. Now one of the main purposes of *Principia* was to create an exact logical language. In other words a machine. However, what makes for its success as a precise language, makes it impossible in everyday life, where ambiguity and vagueness are inescapable components in a language, which (unlike a strictly formal language) can talk about itself. And of course this very ability is also the source of its seductiveness and the subsequent confusion it sows. Although, of course, he does not say so openly. He points out though that the ambiguity of natural language is not something to be deplored, on the contrary, without it, true communication would be impossible between people, who by necessity have different perspective and hence imbue different meanings to the same words.

What is the world? It is filled with particulars and facts. Those lie beyond the realm and control of men, and exist independently of him. We appropriate particulars by giving them proper names. Particulars are limited in space and time, Russell, somewhat arbitrarily limits duration to a few seconds, and is non-committal as to extension, but from the spirit of his account, a particular can be held in your hand. The typical proper name of a particular is 'This'. And a typical particular, at least for a lecturer on philosophy, is a spot of white on the blackboard. Once again, and the point is obvious but easily overlooked. A name is not the same as the particular, it is just a symbol for it. Facts on the other hand are different, they concern particulars, but are not really particulars. Facts correspond to propositions, but unlike propositions they are neither true or false, they are just facts. Facts belong so to speak to the real world. Propositions are symbols, but they are not names. Names are just names, but propositions are either true or false, and it is the corresponding fact 'out there' which determines which. Then there are propositional functions. They have variables, for which proper names of particulars can be substituted making them into propositions. A propositional function is neither true nor false, it only becomes such in particular instances. However, one may say some general things about a propositional function. It can be true for all substitutions of relevant particulars, then we say it is necessary. If its negation is necessary we say it is impossible, in other words no particular satisfy it. Or there are some particulars that satisfy it, then we say that it is possible. In this context we should note that 'all' does not mean that there is some instance of it. This is the paradox of the empty set. All pigs that can read can also fly. If this is not true, than there is some pig that can read but not fly, and as we reasonably assume that no pig can read, the statement is so to speak empty verified. This is something the ancients logicians did not really appreciate, to Aristotle 'all' implies the existence of some. It is in retrospect remarkable that such an obvious state of affairs would have been overlooked for so long. Russell notes wryly that Leibniz, one of the supreme intellects of the 17th century, found it a stumbling block and was not able, due to an exaggerated regard for

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interval, a large part of the thinking is irrelevant digression. What is left is a set of measure zero, but which nevertheless may lead to progress, as the Devils staircase illustrates.

the ancients, to transcend it. Thus it is not true that pure logic has not made significant advances since antiquity<sup>4</sup>.

Now Russell denotes propositional functions that assert something for all to be general propositions, and those that assert existence, that some particulars satisfy, as particular propositions. The crucial point is that particular propositions alone can never imply a general proposition. We cannot prove them by induction, because in the words of the title of Karl Popper's autobiography, that would be an unending quest. A single counterexample does not prove some general proposition without some tacit general proposition. A single counterexample to a mathematical hypothesis, does not imply the general propositions, that all purported proofs will be deficient, unless we assume the general proposition that mathematics is free of contradiction.

Now Russell's contribution to logic is to introduce (many-placed) relations. The simplest kind originally considered was the relation only containing one member, i.e. that of a substance having a certain attribute. 'Socrates is mortal'. This was, at least according to Russell assumed to be all there was to it, and that all propositions basically was of that kind. But what about 'x is left of y' or 'x is between y and z'. And so on. There is no limit to the number of places to be filled in a relation. A 'n-relation' can always be reduced to a 'n+1-relation'. Instead of saying that 'x is yellow' one may have the more abstract 'x has the same color as y'. Now this is, as we all now know in retrospect an equivalence relation, although at the time this might not have been quite as elementary an observation as nowadays. But of course we may simply define yellow to correspond to a particular equivalence class<sup>5</sup>. Now this reduction has been anticipated by Berkeley and Hume, but they were under the mistaken assumption that by so doing they were getting rid of abstract ideas altogether. Not all '2-relations' occur as reductions of '1-relations', only those which are equivalence relations.

Now, how do we make sense of a proposition? We make sense of it out of the words of which it is composed. If we do not have the vocabulary we are stuck. Individual words are arbitrary symbols, and what they mean we cannot convey by words, only by being intimately connected by what they refer to<sup>6</sup>. But if you know the words, and are familiar with the conventions of syntax, you can infer the meaning and understand sentences which you have never encountered before. We do it all the time while we are reading. Some words are of course easy to understand. You can simply point. When it comes to abstract words, you can no longer point. Somehow we sense their meaning anyway. Maybe because we feel the need to have words for such meanings. Still, it is this that so easily can let our

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<sup>4</sup> One may refer to the old sophist reasoning which I encountered as a child. 'A cat has one more tail than no cat. No cats have two tails. Thus a cat has three tails. This was of course never meant to convince anybody, only to make fun of formal reasoning. A more direct illustration is the syllogism.

<sup>5</sup> If you cannot distinguish red from blue, do you see 'red' or do you see 'blue'. This is stupid David Mumford reminds me once. Colors are really equivalence classes. If you think in this way the problem disappears. You simply note that what is supposed to be blue looks exactly as what is supposed to be red. What it really looks like does not make sense.

<sup>6</sup> This is a common sense point of view. And common sense often goes a long way. In particular it makes dictionaries possible, in which words are explained by words. Of course words could also refer to linguistic constructions, in other words symbols seen as particulars, to this we may return.

thinking go astray. There are words that are hard to point to, connecting words such as 'and', 'or', 'either'. They connect different propositions. There is an elegant way of making their meaning precise, namely by truth-tables, supposedly the invention of Wittgenstein<sup>7</sup>. One may here be tempted to become a bit technical. Given two propositions  $p, q$ . There will be four combinations of truth and falsehoods among them, namely  $TT, TF, FT, FF$ . To each of the four possibilities we assign either a  $T$  or an  $F$ . There will be sixteen possible schemes. The connector 'and' correspond to  $T, F, F, F$  while 'or' by  $T, T, T, F$ . Those sixteen possibilities may naturally via binary representations be encoded by the numbers  $0 \dots 15$ , say 'and' by 8 (1000), while 'or' by 13 (1110). Similarly 'imply' by (1011) i.e. 11. Most of the numbers do not correspond to anything reasonable. A nice technical problem presents itself. Can you express all of those sixteen possibilities by using only 'and', 'or' and 'imply'. Or even fewer if you chose some non-standard one. Russell refers to the notion  $p/q$  as being incompatible which obviously correspond to 7 (0111) and claims that this will generate<sup>8</sup>. Is this philosophy, science or mathematical combinatorics. Obviously the latter, but does it have philosophical ramifications? Is this the kind of logic we would like to engage in, solely on the level of symbol modification? Still there is some undeniable charm in such a play. The vagueness of philosophy is replaced by something precise and amenable to manipulation, leading perhaps to some unanticipated thoughts.

Russell makes a big deal by making a distinction between proper names and descriptions. Those should not be confused. A description may or may not be satisfied. It is in a sense a propositional function. If it does not identify an individual it cannot be equated with a proper name. To say that 'Scott is the author of Ivanhoe' is not the same thing as saying that 'Scott is Scott' or 'the author of Ivanhoe is the author of Ivanhoe'. Both of those are tautologies and tell us nothing about the world beyond that of logic. To say that 'Scott is the author of Ivanhoe' is the same as finding a specific particular, related by its symbol namely its proper name, which uniquely satisfies a description. Now, Scott may not really be a particular, not in the sense that Russell interprets the notion of a particular to be, but that is another matter to which we will return.

Now the deeper reason for making this distinction is to do away with much silliness that is associated with existence, which sometimes has led to the sub-notion of existence known as subsistence. Just to say that 'A round square does not exist' somehow grants the 'round square' some provisional existence, subsistence if you want to label it with a word, in order just to say something about it, even if it is just that it does not exist. This is stupid according to Russell who proceeds to try and clear up the mess. First and foremost, existence is not an attribute. It is pointless to say that 'Socrates exists' if 'Socrates' is thought of as a proper name. A proper name refers to something, and hence claiming its existence is redundant. Now 'A round square' is a description, to say that it does not exist, means that the description as a propositional function is impossible. You cannot find anything that satisfies it. In the same way 'God' is not a proper name, it is a description. If 'Jesus' is a proper name, 'Jesus' does of course exist, otherwise it would not be a proper name. What does the statement 'Jesus does not exist' really mean? For

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<sup>7</sup> One is surprised. Did they not appear long before, if not by the hand of Aristotle at least in the writings of the Stoics

<sup>8</sup>  $p/q$  simply means that  $p$  and  $q$  cannot both be true. We can then e.g. define  $p \mapsto q$  as  $p/(q/q)$

there to be a meaning at all, 'Jesus' needs to be replaced by a description. There are many choices for a description. One would be that 'Jesus was crucified, revived on the third day, and eventually went to heaven'. Most non-believers in Christianity would reject that the possibility of this description actually being satisfied, and thus find it rather uninteresting. There are alternative descriptions which makes the question more interesting, such as relating to a possible historical person with no super-natural attributes, but with some of those mentioned in the gospels.

Thus to iterate, and Russell sometimes goes out of his way to iterate a statement several times in order to get a point across<sup>9</sup>, existence does not belong to objects. The notion of existence only makes sense with regard to a propositional function or a description. By exhibiting an object that satisfies the conditions, we show existence. Not of the object, because exhibiting the object is the same thing as exhibiting existence, but of there existing objects that satisfy. To put it bluntly. The question whether Sherlock Holmes exists is meaningless, what is meaningful is to formulate a number of different descriptions which may be satisfied by the notion of Sherlock Holmes. Such as fictional detective invented by Conan Doyle (a proper name) living on Baker Street. While a person of flesh and blood resident at Baker Street at the end of the 19th century, will have many objects satisfying it, but not Sherlock Holmes.

Now what is a particular? Is a desk (another one of those concrete entities in the real world to which philosophers, for obvious reasons, make repeated references) an entity? Is Picadilly an entity? Is Bertrand Russel an entity, as opposed to a particular. What constitutes identity over time (and space)? This is a classical philosophical problem, and maybe in all of those examples, it is ultimately a question of convention. Russell refers to a series. A series of particulars which are connected to each other. This is more than passing fancy, in an attempt together with Whitehead to do for the physical world what they had already done for the mathematical, they try to address the fundamental problems of making sense of basic physical concepts such as matter, out of basic events in space-time. The question is one of convention, on the other hand it is no mere formality. We are less convinced of identity when we meet a stranger twice. Is that really the same man we saw a week ago? In the case of a stranger there is little we can go on. When it comes to ourselves there is so much more. We have our thoughts and memories, to start our with, and our sense of inhabiting a body. Related to this Russell makes the commendable assertion that a hallucination and a phantom are as real, as particulars, as anything else in the world. To say that a hallucination is unreal is simple just a term of abuse<sup>10</sup>. What is true that a hallucination is not connected with other particulars we expect to accompany it. Something 'real' we see in front of us we can also touch, and the tactile sensations mesh nicely with the visual. Thus the 'real' thing in front of us is more coherent and more of a complex entity than the hallucination, but of course the hallucination does exist nevertheless as a hallucination. If you would be asked by your psychiatrist whether you have hallucinations, you can of course deny them, but that would

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<sup>9</sup> This actually being transcribed lectures, it makes sense.

<sup>10</sup> A non-existent object can never provoke our anger to the extent that it would make us abuse it. A non-existent thing can do nothing. Non-existence on the other hand may frustrate us. But non-existence does not correspond to some specific object which does not exist!

be a lie. The hallucinations themselves are facts which would render such a proposition false.

Now what do you have in the world? If you would like to make an inventory, what would you list? You have particulars, as the nominalists would agree on. But once you have particulars you can form sets of particulars, and those should also be entities of the world. And once you have them you can start building up sets of those *ad infinitum*. This of course leads to the Russell paradox on which the author dwells at lengths. Not directly of course, but from the contradiction that on one hand, the number of subsets you can form out of a set exceed the cardinality of the set itself, as elegantly proved by Cantor. While on the other hand if you make the set of all that exists, it clearly contains everything, and its cardinality cannot be superseded. So what do you do? You analyze Cantor's proof and come up with the Russell paradox of the set of all sets which do not contain themselves as members. This is clearly an illegitimate construction. Why? Ultimately it reduces to the liars paradox stemming from antiquity. It involves self-reference. By introducing the hierarchy of types, Russell makes such self-references impossible. To most of us it seems somewhat contrived and merely a technical subterfuge, yet it ties down with the notion that the world is not given to us, we build it continuously, and as we do so, references changes as well as meaning of words. Thinking creates what was not there before. Thoughts that have never been thought before have not existed as thought, even if there would be a ready made way of encoding them when they are thought for the first time<sup>11</sup> Russell takes his theory of types seriously, no wonder as it saved him from an impasse in which he had been imprisoned for years. In particular he claims that each type has its own relation to reality.

Russell is skeptical about philosophy. While in science there is a large consensus, in philosophy there is almost none. As hinted at above, one gets the feeling that Russell regretted his choice of career. Were there not so many more interesting things going on elsewhere? What value would his philosophy have? There are so many subtle things to think about, and the real important things are so painful to focus your mind on, so you can only do it for a very short attention span, as already noted. How easy is it not to fall into an unsuspected trap? Can one ever really been sure of not having made some mistake? In science, and also in mathematics, one get empirical confirmation<sup>12</sup>. One particular test for his careful analysis of language would be to judge the veracity of the theory of neural monism, put forward by William James. While he was scathing about the pragmatism of James he was far more respectful about neural monism, which explained the difference between the mental and the physical, by considering two different listings

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<sup>11</sup> In the well-known fable, the Library of Babel, Borges imagines a very large, but finite, library of all possible books. Such a library does of course exist in various incarnations, such as encoded as a vector space of some finite dimension over some finite field, or simply as the numbers from 1 to whatever it takes. But of course they do not exist (as yet) as real books, which are not just sequences of characters, but entities imbued with meaning by those who read them. Of course one may make it all into mathematics, as above, or even meta-mathematics, by considering formal languages (in particular computer-programs), out of which some non-trivial insights can be derived out of such thought-experiments.

<sup>12</sup> Yes, Russell agrees that the deductive method is not by itself sufficiently powerful to impose belief. In fact we are not so much impressed by the self-evidence of the starting principles, as being reassured by the consequences of the process

of the same objects (say addresses of customers, according to geographical location or alphabetic order). The two lists maybe very different and serve very different purposes. Yet, how do monists deal with wishes and beliefs? Do they really imbue them with a real independent existence? Neural monists tend to take a behavioral approach, only the external exist, which of course goes against common sense. On the other hand he admits a certain partiality for the theory, if for no other reasons than aesthetics, based on Occam's razor.

Now the subject matter and approach of Russell now strikes us as a bit quaint. His naivety when it comes to have some spontaneous familiarity with an object, may be contrasted against Wittgenstein's view that in order to fully understand the meaning of an object, you need to see it in all its contexts, which may of course be an infinite number. Much of his motivation is of course to use language for greater clarity and to resolve many classical philosophical problems as pseudo-problems, very much in the spirit of the latter Wittgenstein. However, Russell took exception to the second incarnation of his student. His notion of languages being not vehicle for understanding, but as games to be played. In the words of Russell, mere parlor games. There are of course innovations which we now take for granted. The notion of truth tables, which may have been thought of as striking at the time, is now part and parcel of the most elementary courses on logic, and high-school students encounter them nowadays (or ought to). Similarly the notion of an equivalence relation is so elementary that it is hard now to believe that it once was pause for thought.

To end on a slightly more up-beat term. What pleasure did Russell really seem to obtain from philosophizing? Once clue may be gained from his remark, that in philosophy you should start from assumptions and observations so basic that everyone wonders why you even bother with them; and then end up with something so paradoxical that no one will believe it.

April 5, 2011 **Ulf Persson:** *Prof.em, Chalmers U.of Tech., Göteborg Sweden* ulfp@chalmers.se