Maths, Games & Recreations

An Intimate Connection

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On the face of it is yet another collection of mathematical recreations, but it is more, much more. Ostensibly it is a philosophical argument about the nature of games and mathematics, what they have in common and in what ways they crucially differ. Although the author does not say it, and may take exception to it, I consider him a Platonist. Rules of a game may be up to the discretion of their inventors, but once invented it has consequences beyond the intentions of the inventors. Games are highly abstract things, and they do not reside 'out there' in the physical universe of space and time, although of course there are various physical manifestations of them, but are mental objects, not confined to the brains of single individuals but are universally shared. The author makes the nice point that while physical objects are unique, our mental conceptions of them vary from individual to individual; while games are the same mentally, although their physical manifestations vary widely. Of course a game is more than its set of rules, with the set of rules there is a hidden assumption that there is such a notion of a game, that the rules are there to be followed, and the point of the game is to win. Of course those basic assumptions, without which a game would never take off ground, cannot be decoded in the rules themselves. You cannot make 'follow the rules' as one of the rules. Thus would we encounter a foreign intelligence it might not be so easy to share a game of say chess; although from the human point of view as the author points out, games transcend cultures in a very striking way. They appeal to a very basic instinct in us, present in particular in children.

The rules are just the beginning, you cannot have any idea of a game unless you play it. By playing it secondary notions will invariably emerge. The more interesting the game, the more people attracted to it in a serious way, the more elaborate will this secondary structure become. The sources of those structures are given by the mental constitutions of the players, their actual experiences, and the constraints given by the rules. Three strands which are a bit difficult to wholly separate. It is possible that a foreign intelligence would develop an entirely different superstructures, just as it is possible to imagine that our present superstructure is a historical accident; but the interesting thing is that the outcome of an actual game is independent upon the structures. It certainly is not a case of claiming that who wins or loses depends on your point of view. While the secondary superstructures are different we may actually in an objective way ascertain which of us or the extra-terrestrials are the better players.

The author refers to the secondary superstructure as the science of chess. I think this is a good analogue. To understand the playing of chess you need experience and illuminating notions and concepts. What is a weak square? It is not part of the rules but it is an emergent feature which a science of chess is bound to pick up. Or is it? Would a foreign intelligence develop a totally different science of chess, creating other types of concepts? We can only speculate.

Mathematics is a game, but also much more. It is a game in the sense of being rule-bound, and that the rules somehow creates its own world in which we discover things. Mathematics shares with games the same kind of mental type ontology, it is beyond space and time so to speak, and as far as it is mental it is a public mentality, beyond the whim of the individual. But Mathematics is different from games in a very profound sense. Not just because games are competitions and would not manifest themselves without the encounter of two opposing wills, but because somehow games are limited and dead-ends. Some games can be reduced to mathematics, and once they have been so explained they somehow lose their interest. Once they have been 'solved' there seems to be no space left to the imagination, they become if predictable and hence boring. But mathematics is a super-game that cannot be reduced to itself, which Hilbert were to learn. With computers some of the human thinking has been simulated on the computer to the extent that nowadays the grandmasters of the world are chance-less against Deep Blue or whatever they are called. One wonders whether this will make chess less interesting, even if it has not been reduced to mathematics, it has been reduced to a much larger brute force application, against which even the strategic intuitions of the best players in the world are chanceless. In the case of Go success in simulation has been far more modest and thus it may maintain its fascination for much longer. But regardless of the ultimate triviality of games, which are sufficiently finite to allow computers to master, games are dead-ends, they do not extend beyond themselves and they certainly have no applications. Mathematics on the other hand, while locally being very much of a game, in fact many local arguments show striking similarities with the playing of games, and many proofs are in the nature of solving a local game, it transcends it. First because of the mysterious interconnectedness of mathematics were various sub-disciplines which seem totally disjoint almost always throw light upon each other. This mysterious interconnectedness of mathematics is connected to what I would like to call strong Platonism. It is a Platonism that goes beyond the mere mental ontology it shares with games. It is an indication that mathematics is not some pure accident of rules, but it is something divine which mankind has stumbled upon. The great mystery is that so many mathematical questions do indeed have accessible answers. A fact that fueled Hilbert's optimism expressed via 'we need not know, we will know'. Mathematics has also applications to the real world as well as being very much inspired by it. There has been other purer mathematical traditions, and the author refers in particular to the Wasan tradition of Japan, but they seem to have eventually petered out. Mathematics transcends games because the rules of mathematics are not arbitrary and also subject to flux, in spite of the fact that mathematics is often portrayed as a deductive science, in which axioms are set at the beginning and then consequences are churned out. In many disciplines the finding of the right definitions and assumptions take time, while others, such as group theory, are fully grown when they appear. Also mathematics is like language in the sense that it contains its own meta-structure. A mathematician will ask different questions about chess than a chess player. The chess player is only interested in improving his game and win. A mathematician asks reflective questions. How long can a chess game be, how many games are there, what happens if we perturb the rules. Axiomatizing a piece of mathematics is akin to making it into a game, but then there is the meta-question is the system consistent? The system may be so abstract that its objects may not signify anything concrete at all, but as system it is a concrete object codified by strings of symbols and hence in principle becoming numbers and translated into a question of number theory, however contrived.

Mathematics is not about calculation, although calculation by itself can be rather exciting, it is about imagination, creativity and play. The same kind of play and imagination employed when playing a game. The purpose of the book is to show and illustrate this taking as the point of departure games, to which the public has an easier time relating. Maybe children who are naturally excited by mathematics sense this connection right away, while those to whom it remains a dull incomprehensible subject have simply missed the connection. Thus a mathematician in work is playing, and his imagination is constantly being challenged and hence stimulated, because it is not total freedom that makes the imagination flower, it is constraint. The mind that meets with no barrier but to which everything is pliable is not stimulated. The author explains that the particular excitement exercised by mathematics, is due to a having expectations both met and frustrated. There has to be a certain predictability, otherwise you would have no control, but there has to obstacles that force you to maintain your control. The phenomenon is not that different from music, the pleasure of which is derived from a mixture of confirmation and surprise. This does not mean that there really is any connection between mathematics and music, except the formal ones enunciated already by Pythagoras. The author is careful to point out that many mathematicians are not musical, and that one hardly expects a musician to be mathematically gifted.

The author has decided to illustrate the nature of mathematics by examples. This is no bare compilation of such, of which there are many, but the presentation is instructed by his overall mission. The choices are of exquisite taste, and although much material inevitably overlaps with standard collections, there are enough original examples to make the enterprise unique and personal. Even a professional mathematicians finds much to delight. The examples show that even if you may enter mathematics in a game-mood, and play is an essential element, the pursuit of mathematical problems opens up unexpected doors. Just as in the case of games, mathematics is not a spectator sport. You cannot read mathematics and be instructed, as little as you can learn to ride a bike by reading a book. You need hands-on experience. What is important is to make mistakes, and only by making mistakes do you get an understanding of what it is all about, why the easy and immediate approaches do not work. This becomes part of so called tacit knowledge that is impossible to write down and codify and thus transmit.

The author ends up with a discussion of personal idiosyncrasies, while some mathematicians are more game-oriented than others and crave clear definitions and rules, others are more flexible. There is also a difference between geometry and algebra, although one should in my opinion not make too much of a distinction, because most of modern geometry does not really allow direct visualization in the way plane and to some extent 3-dimensional geometry allows. It is clear though that a visual presentation of an idea often allows you to see something in a flash (and admittedly even higher dimensional geometries allow suggestive visual analogies), while algebra is more of a sequential thing.

Long computations may be reassuring as verification is, but they are seldom enlightening, the way a visual presentation is. This leads to the important question about the purpose of proof. In simple games verification can often be done by a case by case analysis. But verification seldom explains. For more complicated situation you need to find an underlying reason. Thus the purpose of a proof is not just confirmation that something is true, but it is also an attempt at explaining why something is true, and often arguments invoked in a proof often have a wider applications, and thus enable you to generalize or to apply to a different situation. Proofs are no mere mechanical contraptions that are dutifully performed, they are a way of presenting mathematical thinking, that often give new ideas. On the other hand reading a proof may be a mechanical process, in which there is so called local understanding, meaning that the reader is able to follow each step but have no feeling for the ideas and the intentions. This is often due to the tradition of presenting proofs in a terse and efficient way in which subjectivity is banned. To understand a proof you often need motivating ideas, explanations of why the author does this and that, which is often due to some previous failed reasoning of which he or she is careful to hide. As already explained, you learn by making mistakes. Presentation of mathematics seldom presents mistakes, those are left for the reader to make.

Mathematics is a wonderful adventure, maybe intellectually the most exciting available to man. The spirit in which it should be undertaken is pure curiosity and for an appreciation of beauty, which the author reminds us, is not an add-on or a mere decoration, but is integral to the whole mathematical experience. Consequently he abhors modern fads about cost-effectiveness, that certitude comes at a price.

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