The Math Instinct

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This book has two purposes. One, no doubt imposed by the publisher, is to give a pep-talk to people who are uncomfortable with mathematics and assure them that they are indeed much better than they think, there is nothing wrong with their intelligence at all, and they do possess abilities which are more than adequate to deal with real life situations. The second, no doubt providing the inspiration for the author[1], is to give a list of some intriguing mathematical phenomenon which can be observed in the natural world. The two themes are of course not unrelated, only the emphasis and the interpretation.

First animals, even insects, exhibit remarkable abilities of locomotion and navigation. How do they do it? One standard explanation nowadays is that they have evolved such proficiencies through natural selection, and thus that those are hard-wired into their brains. However, this explanation does not really probe much deeper than the old one that Nature designed it accordingly, or that God in his wisdom and benevolence arranged it this way. It is seductively easy to design scenarios retrospectively that will explain almost any phenomenon. When it comes to natural selection one should keep in mind two things (maybe even three if one probes beyond classical selection) that the environmental impacts on the development of a species come in different orders. Some are indeed very direct and have a drastic influence on reproduction, others are very indirect and are only related to reproductive fitness through a long chain of contingencies, and consequently their effects, if any, can only be manifested over a very long evolutionary time interval. Secondly that evolution remembers by genes, at least in the modern synthesis of Darwinism with Mendel, thus any feature evolved evolutionary is in fact reducible to a gene, or rather a combination of genes, and hence there would be in principle a biochemical explanation. No such biochemical pathway has ever been established, at most various fragments of such have been exhibited. This does not mean that they are none, only that it might be humanly infeasible to present such explanations[2] And thirdly, not all manifestations of organic behavior have genetic roots. An animal that tumbles to the ground does so because of gravity, not because of genetic determinism. In fact in the embryological development, purely physical processes play a role. Thus in particular finger prints of identical twins differ, and conjoined twins exhibit many features, such as the intricate ways organs are shared, which are not genetically determined as opposed to constrained. Related to this is that any invention has features that are not consciously designed, and similarly an organism may have emerging features which are unrelated to any specific evolutionary pressures, which make

[1] and according to him left-overs from his previous book - the Math Gene, which goes over pretty much the same material

[2] Fermats theorem, or maybe more to the point the classification of groups can formally be reduced to long chains of arguments, in the second case too long even for mathematicians to fully fathom. Detailed biochemical 'explanations' may similarly be beyond the ken of human global understanding.

the retroactive explanations so speculative and unreliable (Just-so stories). S.J. Gould refers to those as corbels. And finally related to the caveat of orthodox explanation, it is not only genes that are transmitted from generation to generation, although that provide the strictly biological component, culture, in a very extended sense is also crucial to the survival of species. The bacterial fauna with which higher organisms live in symbiosis, is a case in point, transmitted to the progeny through intimate contact. Thus even would we be able to recover the DNA of extinct species we would not be able to bring them to life, because crucial traditions have been broken and cannot be recaptures as there is no encoding for them.

So how do animals do it? Humans can simulate their accomplishments through mathematics. Does that mean that they do mathematics? Devlin does indeed argue so, although of course he does not claim that they do so intentionally and consciously, only that the brains are hardwired to do the necessary steps (whatever they are), just as a computer is made to make complicated calculations at top speed. The brain as a computer is a common metaphor in the sense of Manin, and certainly many of the mechanical steps of an organism lend themselves very well through computer simulations via often simple algorithms. How do we keep our balance on a bicycle? Clearly humans have not evolved to ride bicycles, so the ability to ride them depend on some less specialized skills. Any balancing skill clearly depends on a very quick ability to readjust movement based on quick feed-back, not unlike standard ways of solving differential equations (partial or ordinary) through successive approximations. If you like you can simulate the process by setting up appropriate equations, the organism does not need to formulate those, it is enough that the appropriate neurological mechanism is there. And although the neurological mechanism maybe easier to explore, than the biochemical, this has rarely been done so far in any explicit way. Then do planets do mathematics when they orbit the sun? In that case simulation coincides with the real thing. It shows one should be careful, much what seems to be an organic calculation, is revealed on closer scrutiny to be the outcome of physical forces. The spider that weaves its web, actually follows some very simple steps, and then gravity and other inanimate forces do the rest. And of course many intricate mathematical structures are generated by very simple principles. To the untrained eye it could be a minor miracle that such structures could evolve from almost nothing. There simply does not seem to be enough information. Natural evolution is at least metaphorically such a phenomenon. It proceeds through a very simple principle, in fact the philosopher Dennett claims that it is an algorithm. The algorithm of natural selection is of course simple in principle, but it is not a technical algorithm that can be articulated in a precise way in a precise context (it is a metaphor after all). And furthermore one may argue that it feeds on the information rich environment.

Some of those stories are simply very generic, in other cases one can actually through sophisticated experiments obtain very specific detail. How do bees judge distance? It seems as if they do it by visual clues obtained from their movement. Thus if they are forced to fly higher than usual, they will fly longer than necessary (prompted by the information given by the bee-dance), because to them they will appear to fly slower. Similarly if they are forced through featureless tunnels that will confuse them likewise.

Human vision is a well-studied subject, and one which we all have intimate knowledge, and where the action of the brain can be made very manifest by so called 'optical illusions'. Stereographic vision is one obvious feature. It is conveyed through a variety of ways. For close-up objects the movements of the eye-balls and the accommodation of the lens[3] provide the necessary information, then more generally parallax works well at intermediate distances, and that is usually the process with which we actually connect depth vision, especially the sense of depth popping up, rather than being an intellectual awareness. Various tricks of stereographic illusion are based on parallax[4] . Movement also gives parallax without the need of two eyes, although the effect of depth is not so direct but rather in my opinion inferential. The upshot is of course that visual processing, which we all do effortlessly, involves very sophisticated mathematics. And not only humans do high-level mathematics, even lowly animals do. If that is so, why be impressed by mathematics? It is a natural thing in life, but surely we humans are better than that, what we are good at is not mindless calculations, although when it comes to the subconscious variety we can hold our candle, but higher order things, such as seeing patterns, recognizing meanings, reading the thoughts and intentions of other people. In fact it is our intelligence that makes it so difficult to us to perform arithmetic operations, in particular to memorize the multiplication table. The conclusion is inevitable. To do mathematics is trivial and unnecessary to boot. And those who persist in doing it, even being good at it, are nerds, and surely mentally deficient and lacking in imagination and creativity.

This is of course not what the author really thinks. After all he is a math nerd himself, and between the lines his enthusiasm for mathematics shines through; but in his willingness to bend over backwards, by supplying new supporting evidence he certainly plays up to well established preconceptions of mathematics. Mathematicians are notoriously very bad at public relation, so bad indeed that they cannot even reach the public. Devlin has on the other hand been very good at it, and he has done a great service for many years, not only in writing a lot of popular books, but what is even more impressive, regularly contributing to newspaper columns. Such an activity, especially in mathematics, inevitably involves a certain 'dumbing-down', and yes a leaning-backwards, all in order not to antagonize a public with any hint of arrogance, something mathematicians are liable to project, notoriously combining being arrogant with being shy, a disastrous recipe for communication. Thanks to his efforts Devlin has put himself in a position in which he can actually make a very positive impact, and although his intention surely is to liberate mathematics from the vulgar conception of just being a matter of doing sums, I am afraid that the average reader will have that impression confirmed. How much more successful are not the physicists in self-promotion. No bashfulness here, although a lot of arrogance. And it works beautifully. What is the pinnacle of intellectual achievement in the public mind? The answer is easy, you only need to say 'Einstein'. True something of Einstein's

[3] For those who still maintain the ability.

[4] The author brings in particular up the modern invention of autostereogram, a primitive version of which I remember from childhood in sleeping compartments in trains. The ceilings of such are usually perforated by lattices, and by staring at them you can make the ceiling recede or come down towards you, with practice you can even see many different levels of ceilings. The explanation is straight forward due to the regular pattern of the holes which may fool you as to what spots are identical. The effect is very striking.

glory also rubs off on mathematics, so when the physicists keep treating mathematicians as poorer cousins, they are still their best supporters, inadvertedly or not. (And of course mathematicians pay back by always trying to ultimately justify their subjects by claiming that they have physical applications, as if this was being justification enough.) People who are bad at mathematics, should not be able to exult their incompetence, but should be ashamed. After this digression on the public relations, I can return to the last subject matter of the book, maybe the most interesting after all.

Street kids in Brazil are very good at mathematics, meaning at doing sums. They have to learn it in order to survive in their jobs. Such things are not very demanding, in fact far less demanding than many other feats we take for granted, and sure enough being forced to do it, they learn quickly. It is interesting to watch their performances, their calculations are done in an *ad hoc* manner without the benefit of standard procedures but invariably accurately[5]. However, when it comes to pencil and paper work, the situation is completely changed. This is actually a test situation and they expect to do 'school-mathematics' and do miserably, mindlessly following algorithms the nature of which they do not understand on sums that mean nothing to them. What conclusion to draw from this? That mathematical instruction in school is pointless for most students? Or that the instruction should be radically changed? And if so how? And with what expectations? A similar, but not as interesting example, is given by supermarket shoppers comparing prices. The conclusion is not too surprising, after all the skills required are rather modest and people should have no difficulty acquiring them. Is mathematics an artificial cultural artefacts, or rather is elementary calculations so, possible to learn only by counterintuitive effort? People acquire the mastery of their mother tongues as a matter of fact, why not elementary mathematics as effortlessly? Because it is not hardwired into the brain? In fact doing numbers seems to be intimately connected with language, in fact we hear numbers, and most people being fluent in other languages, still do their arithmetics in their native. Because it is so intimately related to it, or that the initial effort was so exhausting that it left an indelible connection?

Doing serious mathematics is to a large extent not a natural activity, it involves the notion of symbolic concepts, and is done consciously and laboriously. According to the author the abstract notion of number is of fairly recent vintage. It is being predated by trade and the written language itself. This is somewhat hardy to believe but the story from Sumeria is charming. To keep a record of trade small tokens representing the wares were put inside wet clay and then folded into a pouch and dried up. Then in order to inspect his tokens, the trader had to break open the pouch and inspect. This was clearly inconvenient, and so the practice of making marks outside the pouch was instigated, and after a while it was realized that this made the tokens superfluous. A cute story, but I suspect just one of many. The abstract representation of number must have been made repeatedly in human history, maybe even on the individual level. Those discoveries would be made more and more often and then fused into a tradition.

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[5] It reminds me of my own playing with numbers as a small child. Doing naturally and uninstructed.