The Road to Reality

A complete guide to the laws of the Universe

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There is the physical world, out of which a tiny part make out mental worlds, out of which a world of Platonic ideas emerge, mathematics being just a small part of it, and physically relevant mathematical theories a smaller part of the mathematical whole, yet that small part is in principle enough to generate the whole of the physical world. This presentation of a mystery and a magic, whether parallel to or derived from the Popperian-Ecclesian theory of the three worlds, lies at the heart of Penrose book.

Philosophers, humanists and proponents of social Sciences may address the general public presenting their ideas shorn of technical jargong yet essentially unadulterated from one sapient being to another. This commendable tradition of popularization also held sway during the 19th century among Natural Historians. Darwins epochal book - 'On the Origin of Species' was in fact addressed to a general intelligent audience. In the 20th century this tradition came to an end, as it had already done as far as physics and mathematics long before. How is it that popular books on natural science and mathematics, especially the harder parts thereof, must be reduced to trivilizing vulgarizations, ultimately unintelligible to laymen and experts alike?

Penrose has, unique among physicists and mathematicians, doggedly tried to present to the public serious books on important issues, giving the reader the real thing. The first example was the widely acclaimed 'The Emperors New Mind' in which the author took issue with the prevalent philosophy of Artificial Intelligence (AI) of the conscious mind only being the emergent epi-phenomenon of calculations on the neurological level. The counter-arguments the author presented, although emotionally satisfying, such as Goedels theorem and the invokation of quantum physics, may not have intellectually been completly convincing, but that is of secondary importance, the main point was that the author did not contend himself by merely stating say what a Turing machine was but actually giving explicit examples, and in fact embarking upon a mini-course on logic and computational theory, his enthusiasm for so doing brimming over. The subsequent book 'Shadows of the Mind' pursued the themes of the previous book deeper, although quite not as successfully. One may remark that the former inflated claims of AI, especially its strong form, have been retracted, and many people working in the field now try to distance themselves from it. It would, however, be rash to contribute this development to the impact of Penrose books. Now in this third book addressed to the intelligent layman, a book on which he purports to have been labouring for over eight years, is a monster of a tome at one-thousand odd pages, addressing the fundamental laws of Nature.

When Hawkins wrote his popular book on Cosmology[1] his publisher warned him

[1] The Nature of Time

that every equation he presented on the page would half the remaining readership. The only equation that concurs favour with the commercial publisher is Einsteins $E = mc^2$ (with that very notation). Penrose book is dotted with equations, still the publisher is a commercial one[2], surely only Penrose would get away with it.

Not even Penrose is so optimistic (or naive) in that aspect that he does not realise that this will give most readers serious problems. He does in fact propose that the text be read on four different levels, painfully aware that there are people who find fractions and the operations theron puzzling and incomprehensible. However, one does seriously doubt what a fourth level reader (assuming this to be the least qualified reading) would get out of the book. True the introductory chapters, and the final summing up, are philosophical in nature, marred by no equations; but few readers of that kind would have had the stamina to plod through a thousand pages of technical material and thus eventually chance upon it, unless of course they would resort to the usual technique of reading popular magazines, i.e. from the back to the front. But what would readers on the second and third level get out of it (maybe more than those on the first level?) and how large a section of the reading public would they constitute? As a professional mathematician I have found many a gem in the first half of the book that focuses on the mathematical prerequisites, but decidedly less on the physics parts that follows, but more on that later. As to the philosophy part, I share Penrose Platonist creed, and thus find his ideas congenial and in agreement with mine (possibly I find his treatment of Poppers falsification idea a bit less subtle than I would have ideally prefered). The drawback is that I find nothing new, only having my own prejudices and pet-ideas confirmed, which admittedly constitute one of the pleasurable aspects of reading, but which one should transcend from time to time. The point is that the structure of philosophical ideas, important as they are as to ultimate assessment and guiding, is decidely less intricate than the subject to which they pertain. (Which seems natural, if you think about it, the converse would have been perverse.)

The hardest part of the book and the most central concerns physics. Quantum Mechanics and its ramifications, and the efforts of unification with Einsteins general theory of relativity. Still it is not so easy to draw a line between the mathematical and the physical. Relativity theory can be and is often considered as a branch of pure mathematics, and can as such be thought of as a refinement of Euclids geometry[3]. The formalism of Lagrangian and Hamiltonians can likewise be appreciated as independant structures, as does statistical mechanics and the basic aspects of twistor-theory. A large part of the physics section concerns the ontological problem of quantum mechanics involving various thought-experiments involving splitting beans and hapless cats, and can as such become tedious to the rapid reader. Penrose points out that Quantum Mechanics consists of two processes, one, refered to as **U** throughout the book, is completly deterministic and thus conceptually classical, involving the evolution of the solution according to the Schrdinger equation; the other, the reduction part **R** is the one that leads to the problems of the thorny issues of the involvement of the observer, to which various subtle schemes of interpretations have been applied, notably the Copenhagen one. Many physicists consequently throw up their

[2] Jonathan Cape

[3] The mathematics of Quantum Physics is more sophisticated and more involved, and its physical applications riddled with tricks and leaps of faith, troubling more pedestrian mathematicians

hands in despair maintaining that the whole theory should be thought of as a wonderful formalism miracolously giving the right answers. Penrose attitude is that Quantum theory is incomplete, and does in fact conflict with Einsteins theory, and a unified theory needs anyway to be worked out.

The polemical thrust of Penrose book, and maybe the reason for its writing, is directed at the current efforts of unification, string theory in particular as being the 'only game in town'. Penrose is very critical of string theory, and in the books most graphic sections, he likens the whole enterprise to a group of hapless tourists exploring a beautiful city in search of a beautiful garden, none of them understanding the local language, with no maps available, and with only aesthetic criteria as guidance and the trust in the guru-like intution of their leader. Indeed in the absence of direct physical confirmations, the only guiding ideas being those of mathematical beauty, and this should be noted being a fact of life and not a choice. String theory does however follow in a venerable tradition. Hermann Weyl is reputed to have claimed that if forced to chose between truth and beauty he would choose the latter. Dirac did in fact stick to the beauty of an equation, even when it contradicted experimental results, only to be ultimately vindicated. The same attitude, but maybe less arrogantly displayed, guided Einstein, convinced that the secret of nature was to be found in some simple principles[4]. It is easy to make fun of string theory, not only the proliferation of hypothetical dimensions, but also the proliferation of theories, the desperate involvement of every known mathematical technique. To this Penrose also adds a list of technical objections, hard for an outsider to assess. The interesting point is that in the absence of Popperian criteria, which essentially means a common ground on which to test and resolve differences, the discussion degenerates to the kind that is common in the humanities. However, Penrose has to concede the remarkable elegance displayed, the seeming robustness of the enterprise as witnessed in a series if rejuvenations, the latest of which makes the very notation of string theory obsolete to be replaced by a M-theory or a F-theory, the contents of which are still conjectural, the uncanny intuition of their guru Witten, and above all the striking and mysterious applications to pure mathematics, as exemplified that two totally different calculations have yielded the same answers[5]. Something quite deep is definitely going on, but whether it is physics and has anything to do with the unification of fields (i.e. the gravitational of Einsteins general relativity and quantum field theory) or more grandiously - the theory of everything (TOE), remains to be seen. To this Penrose contrasts his pet idea, namely twistor-theory. He does however admit that the same general criticism that he has levied against string theory could also be applied against his own. It is mathematically elegant, and it is guided, in the absence of more down-to-earth alternatives, by mathematical beauty, and that it has excited mathematicians maybe more than physicists. It should also be remarked that the mathematical interest of twistors, although quite respectable, has not been fuelled with the same magic and unexpected coincidences that has accompanied that of string-theory. Twistors, unlike say Seiberg-Witten theory, remains a marginal mathematical discipline.

[4] e.g. invariance of the speed of light, equivalence principles as in the thought experiment with the freely falling elevator and the bending of light, etc

[5] The calculations of rational curves of given degree on one of the simplest Calabi-Yaus - the quintic in CP^4

The phenomenon of string-theory does cast into doubt the nature of objective science. Science should in principle present statements that can be refutable. Penrose makes a point that this Popperian criteria does not apply to string-theory as experimental testing seems practically impossible. More seriously though such a stringent application for scientific qualification would also disqualify Diracs theory of the mono-pole, that claims that the existence of just one magnetic mono-pole in the universe would explain the fact that all charges are multiples of one. It is true that the existence of such a mono-pole cannot be refuted, because however long (in practice, as in practice the universe is infinite) the search we can never exclude the possibility that it exists somewhere. But the essence of Poppers criterion for falsifiability is not that all statements shouls be of that sort, only that some ramifications of the theory can be found admitting testing. Still one would be, on such grounds, forced to admit that string theory is not a science of physics, mathematics maybe, subjected as such to logical coherence, or meta-physics. But Popper does not reject non-scientific theories as worthless, far from it.

More seriously Penrose discusses the social elements of science, the need to follow fashion, the expense of experimental science, which forces young people to take a pragmatic attitude, to choose strong groups of research, hence making sure that fashion perpetuates itself automatically. The possibility of the advent of a new lonely genius, like that of Einstein, seems unlikely, maybe even impossible.

The unreasonable effectiveness of mathematics in the physical sciences has been a source of wonder. Penrose treats it as a mystery, a delightful mystery. In temperament one surmises he is more of a mathematician than most theoretical physicists, with a well-endowed geometrical intuition, and a prospensity to dwell on mathematical concepts. Hence his predilection for twistors, the subject he introduced in the beginning of his career.

When he writes his mathematical chapters he takes on the attitude of explaining to those who have never before encountered the material. To really believe that you can get things across to the true neophyte is very unrealistic, yet it has the effect of freshness, charming both to the author and knowledgable reader alike.

So finally briefly what new things did his mathematical chapters teach me? The composition of rotations appear intuitively unpredictable, but Penrose remarks, that if one corresponds a rotation to twice the angle, rotations can simply be added as ordinary translations, an observation going back at least to Hamilton[6]. I did not realize that there is a natural way of associating to each point of a world-curve a point on 3-dimensional hyperbolic space, simply by considering the intersection of the tangentline with the hyperbolic sphere, thus in particular representing the movement of the Earth as a movement in that space, explaining the well-known phenomenon of aberration as hyperbolic parallax[7]. Also in Minkowski space a geodesic maximizing lengths rather than minimizing it, illustrating the so called twin-paradox[8] Also, which I ought to have known, intrinsic time for a photon

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[6] There is also an attempt to show in a striking way the non-simply connectedness of $SO(3)$ motivating the introduction of spinors

[7] In hyperbolic space there is always parallax, even for objects infinitely far away

[8] In relativity theory an accelerating system can be distinguished, thus there is no symmetry between the two twins. A twin taken a detour close to the speed of light, will thus return with its internal time much shorter than that of its stationary twin.

is zero, hence its intrinsic velocity is indeed infinite. The regular Lorenzian transformations become very natural when viewed hyperbolically. Thus not necessarily a physical interpretation but a geometrical one is very helpful to give meaning to the Lorenzian formalism. On the other hand the point of Einstein is that physics is geometry.

Penrose is ambitious enough to provide exercises, at three levels of difficulties, each with its own symbol. The problem is that the exercises are presented as footnotes, and the symbols announcing their difficulties are almost indistinguishable, a more conventional system with different number of stars would have been much more useful. As a slight curiosity, Penrose introduces a system of rather involved graphic representation of summation conventions, symmetrization, dualization and other features of tensor-calculus. It is hard to know whether this is done in jest (but if so why?) or whether the author has himself used this kind of short-hand writing in his own career. The pictures, when involved, appear daunting; maybe a useful remainder of how intricate mathematical formulas may appear to the mathematically innocent.

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