

Perfect Rigour

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Perelman is a Russian math genius whose antics have caught the imagination of the public. Maybe not so much because what he has actually proved, most people could not care less and understand no more than nothing of his achievement, although of course mathematics still exerts some mystique and being a math genius is at least a bit intriguing. But because of his eccentric ways and his principled decision to spurn the worldly rewards his achievement has earned. Is he simply crazy, or does he adhere to a code of integrity way above that you normally find in your fellow humans?

Masha Gessen is Russian. She grew up in Russia but left in her teens for the States where she has become a journalist. She might be eminently qualified to report to a general audience on the strange case of G.Perelman, especially as she was exposed to some competitive mathematics herself in her early youth.

Mathematics in the Soviet Union had a lot of prestige, especially outside the country. Second class Russian mathematicians were usually considered first rate by western standards. How come mathematics could flourish in the Soviet society? There are a number of obvious explanations to that. One being the need for a reality check. Soviet Union needed to survive, it was after all an isolated country surrounded by hostile neighbors. It needed to have a strong military. Weapon development is based on physics, especially in the nuclear age. Being of the right physics is far more important than being of the right ideological bent. High-powered physics is impossible without sophisticated mathematics. This is not to say that such hard-core scientists were not persecuted by Stalin, many where, some came to actual grief. However, Stalin obviously thought of the politicians being more dispensable than the scientists. So there was a culture, an intellectual sanctuary in fact, and being drawn into mathematics did indeed provide a way of escaping the intellectual strictures of a repressive society. The explanation makes sense, but as always there are complications at the fringes.

Soviet education was supposed to be egalitarian. This is true. On the other hand achievement, be it in sports, chess, classical music, was valued. Special schools were created. Also in mathematics. To foster and nurture an elite was not altogether strange in the Soviet society. In fact Stalin had in the early 30's inaugurated a hierarchal system in which over-achievers were materially rewarded. All in order to speed up the process of industrialization.

Kolmogorov was one of the most distinguished mathematicians of the Soviet period, He was mainly known for his work in probability theory, but his interests in mathematics were wide and penetrating. In Geshens book we also learn that he was a homosexual, that his partner was in fact another distinguished Soviet mathematician - Alexandrov. Kolmogorov initiated special schools for mathematically gifted children. Also a system of mathematical competitions developed, also on a hierarchical level. Kolmogorov was not only interested in fostering mathematical talent, his charges were also encouraged to be

physical, go on hikes, swim in rivers, climb mountains. And poetry too was not considered out of bounds. Russian mathematicians do indeed tend to be more cultured and well-rounded than their western counterparts. But Kolmogorov's initiative was not universally embraced. It was resisted. It certainly did not mesh well with egalitarian ambitions. And as always one should not discount envy. Stupidity and envy may be the driving forces of social life, and the cause of much of what goes wrong.

Anyway the stage is set for young Perelman. He is an excellent pupil. Polite, considerate, eager to please. And of course brilliant when it comes to solving problems. Gessen speaks to his mentors, and introduces the reader to the somewhat strange hotbed of Russian competitive mathematics at the school level. Similar breeding institutes were to be found in other Eastern European countries, notably Rumania, who also tended to attract the best and brightest minds.

Now there is an emphasis in Gessen's book on competition. This might be somewhat misleading. Competitions were important, but those afternoon math-schools provided more than just trying to solve problems. They provided a general mathematical education. Nevertheless Perelman excelled in competition. That he was Jewish, and especially having such an obvious Jewish name, made for some additional problems. Jews were not cut off from society, but they were subjected to quotas. It seemed bad if too many Jews were admitted to universities or to competing teams in mathematical olympiads. Ultimately Perelman got through anyway, but that was due to his exceptional talents and involved some special pleading at high places. He needed mentors above the level of the coaches. Abramov was a high level coach, with a history of some no mean distinction of his own. This was not uncommon in the Soviet system. Gromov was maybe the most distinguished mathematician who promoted him, and also the one who is most sympathetic to his decisions. In fact supporting them whole-heartedly as testimonies to the integrity of Perelman. On the other hand Gromov has never turned down prizes or the cash awards that go with them. Most recently in 2009 in Oslo, being the recipient of the Abel Prize.

The wall fell at an opportune moment. In his early twenties Perelman was free to travel abroad. He was of course a strange apparition to the western mathematician, with his long hair and curling finger-nails and Rasputin like gaze. And he was not perfect about his personal hygiene either. So he was feted and invited. Princeton was interested in him after a talk. He might have felt snubbed when they asked him to submit an application. Why could they not take a genius such as him sight unseen? Gessen seems to indicate. But was he really so sensitive? Unworldly and overly sensitive he nevertheless exhibited enough savvy to deal with modern western society, renting cars when needs be.

A number of American mathematicians came into close contact with him. He tended to be entirely focused on mathematical issues. When people asked him serious questions, he responded seriously and civilly. When people did not seem to care, he resented their indifference. He was particularly aggravated with his meeting with Hamilton, the one who had initially launched the method of Ricci flows as a way of getting at the celebrated Poincare conjecture many years before, but become hopelessly stuck. Maybe the lack of enthusiasm on the side of Hamilton is understandable, at least from the human point of view, if not the mathematicians. Perelman had a hard time understanding it.

Eventually he posted his solutions on the web.

This caused a flurry of activities. The notes were terse, as to be expected, so the natural question was whether it was correct. After all many faulty proofs of the Poincare conjecture had been presented over the years. One of those who took the issue seriously was John Morgan. He set himself to work out the missing details, enlisting on one hand the assistance of a colleague familiar with the hard analysis of PDE, of which Morgan, as a topologist did not have much experience; and on the other hand Perelman himself. As he later reported to me. When an argument of Perelman seemed obscure and he had not been able to make sense of it for weeks he wrote and consulted the author. Invariably Perelman always turned out to have been correct, and the way he first wrote down the argument, in retrospect the clearest and most effective. When this happens repeatedly you acquire a lot of confidence in your charge. Obviously Perelman knew what he was doing and his presentation really did not contain any gaps, at least never any that reflected a gap in Perelman's thinking. Other people went through the manuscript and made their own reworkings. Some of them, such as a Chinese pair, went a bit further, claiming, supposedly on the encouragement of Yau, that the proof of Perelman had been incomplete and that they had filled in the missing details. This caused a furor in the mathematical world, there was even a write up of it in the New Yorker, by the woman who wrote a biography of Nash, appearing timely at the ICM in Madrid 2006, where Perelman was supposed to receive the Fields Medal. As everyone knows, he declined the offer. The President of the IMU - Ball, went personally to St-Petersburg to plead with him. There he was, having by that time cut off all contacts with the mathematical world, including those of his own institution - Steklov at Leningrad. Ball was received very civilly. They had long conversations. Ball tried to be persuasive. Perelman would not budge. In the end the mission was aborted, and Ball reported very respectfully on his encounter. Naturally it did provoke more press coverage than would otherwise have been the due at the ICM, a congress which inevitably appears very obscure in the eyes of the public.

A few years later there was in a sense a repeat. Now it was the Clay prize with a substantially larger cash prize that was to be awarded. The ceremony was set in Paris in June 2010. It might be the only ceremony to be arranged for the award of the Clay prize this century. It was by then clear that Perelman would have nothing of it.

Are mathematicians strange? It is now very fashionable to assume that the presence of smart autism, i.e. the Asperger syndrome, is particularly prevalent among mathematicians. The Asperger comes in many shades; in fact there is a whole spectrum of it, due to a plethora of questionnaires and tests. As noted mathematicians tend to rate high on them, then comes physicists and chemists, biologists less so. In the humanities the affliction is no higher than in the general population. It makes sense. Or does it? Is this not anything but an attempt at reduction. To trivialize personal quirks and originality, not just to plain eccentricity, but to a syndrome. The instinctive reaction of the mediocrity to spurn excellence and originality of mind, in order to protect itself, intolerant as it is of waywardness and non-conformism.

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