**Science and Hypothesis**

*H.Poincaré*

September 4-6, 2009

The march of Science is towards unity and simplicity. If not there is no science, only a growing pile of isolated facts. Science is about generalizations and predictions, to see the simple structures behind a confusing multifarious appearance. This is a program that can at least be traced back already to the pre-Socratic philosophers, its most succinct expression having been given by Parmenides. The great mystery is that the universe so well adheres to the strictures imposed by our minds, i.e. that it is so simple, simple in a way that is congenial to us.

Poincaré is no philosopher. He is impatient with the meta-physicists, and he does not embroil himself in the petty details, the quagmire of which so allures the philosophical mind. This does not, however, means that the book is not philosophical, only that for such a work it is remarkably lucid, exhibiting a seductive combination of sophistication and common sense. Poincaré has no truck with nonsense, he reasons briskly and always to the point. He is a delight to read.

In many ways Poincaré is a formalist and a pragmatist, and as the latter he fits well in the philosophical tradition of William James and C.S.Peirce, with the added bonus that his documented brilliancy as a mathematician and mathematical physicist endows him with an authority that is not entirely present in the former. He writes as a physicist not as a mathematician, because mathematics is not science, although indispensable in any pursuit of the former. How is that? The naive and vulgar conception is that science is about measurement and numbers, Poincaré obviously sees deeper than that. Not that he disparages measurement, on the contrary, according to Poincaré, to a scientist it is more important to be able to measure something than to know what it 'really is; but because he understands observable phenomena as the superposition of many similar elementary phenomena. In the words of Poincaré, mathematics teaches us to combine like with like. Continuing

> *Its object is to divine the result of a combination without having to reconstruct the combination element by element. If we have to repeat the same operation several times, mathematics enables us to avoid this repetition by telling the result beforehand by a kind of induction.*

Induction that is what lies at the core of the integers, (and on may add that according to Peirce something more basic than logic itself). Induction is a kind of infinite syllogism, graspable by the human mind by its similarity at each step. So Poincaré reasons in the beginning of his extended essay, in which he proposes to scrutinize the pillars of mathematics, arithmetic and geometry. Are numbers real or artifacts of the human mind? Obviously such a basic question cannot settled by even more basic arguments. If pressed Poincaré no doubt would consider it as close to a Kantian synthetic a priori as we can get. Poincaré is no mystic, he does not care to delve further, let us just agree that the mind cannot conceive of anything different, he seems to say, and that is the way it is, and the mind being what we are stuck with in our explorations, we just have to accept it and go on from there.

When we come to the notion of real numbers we leave arithmetic and enter into geometry, although from a formal point of view, real numbers are as any student of mathematics knows, derived from the integers. The continuum is based on spatial intuition. The disturbing aspect of which is that it could very well happen that the locations of three points $A, B, C$ ar such that $B$ is both indistinguishable from $A$ and $C$, while $A$ is different from $C$. This goes against our conviction of the transitivity of identity, forcing us to go deeper than our limited senses allow us to penetrate. Thus the mathematicians real line has no physical counterpart, to claim so would be silly metaphysics, it is but a mathematicians attempt to formally capture this evasive intuition and make it logically impeccable. To bring logical consistency to the information purveyed by our imperfect senses. [1]. Space is likewise a construction of the mind. The notion of empty space is an absurdity, space is something we construct as we move around. Things are not put in space, space is created to make sense of the relations between things. Spatial intuition is something we laboriously build up, integrating visual sensations with muscular, and very much due to our ability to being able to manipulate our environment. In his discussion, he is rather reminiscent of William James in his Principle of Psychology, and for all we know, he might even be familiar with the latter work, appearing some ten years before the writing of the present essay. Obviously Poincaré does not imply that space is a fiction, our conception of space must as some level be congruent with an outer reality, but our conception of space as such is of our own making. One may, I think, fruitfully compare this with our sensory worlds. Our qualia (such as our immediate sensation of say the color red) cannot be exported and thus directly compared with those of others, only more abstract notions such as relations between qualia, maybe transported from one mind to another. The experiences of different minds are not equal, the notion of equality does not makes sense as they do take place in different minds, but they may very well be isomorphic. A concept can be manifested in many different ways, but there is no particular prototype (as Platonism is often misleadingly understood as). In the same way Poincaré stresses that we are not concerned with physical concepts as such, only with their relations with each other. (And thus once concludes, in their amenability to mathematical treatment. A very Platonist point of view in a sense.). The notion that space is 3-dimensional is even that open to doubt. Poincaré died long before virtual realities would become a feasible option, but from his discussion it is rather evident that he would be rather congenial to the experiments such could make possible suggesting indeed that not even the 3-dimensionality of space is something physical, individuals exposed to more complicated inputs, might very well construct a 4-dimensional reality[2]. To Poincaré our conceptions of space are but convenient

[1] I learned differently. When I encountered the mathematical notion of the continuum as a teenager, I took it very seriously to the point of accepting it literally. To me at the time, and long since after (including my professional career), I thought of the real numbers as a true physical entity, especially as to time, worrying whether life was an open or closed interval in eternity, whether there was a last moment of life or a first moment of death.

[2] One wonders as to the possibility of inducing in a child a real tangible 4-dimensional intuition, by

categories of mental organizations[3]. In particular Poincaré makes short thrift with the idea that Euclidean geometry tells us anything about the physical world. Whether Euclidean or non-Euclidean geometry is 'true' is nothing that can be settled by experiment[4]. To ask which one is true is like asking whether it is more correct to measure lengths in the metric system rather than using feet and inches. Just as there is a simple conversion factor between the former methods, everything can either be expressed in Euclidean or non-Euclidean terms, by similar, if admittedly somewhat more involved conversions. It is just a matter of convenience. The idea, as proposed by Lobachevsky, that the lack of any detectable parallax was a case against his astral geometry, is flawed. Either we assume that light moves in straight lines or it does not. This is clearly not a mathematical question that can be settled by mathematics alone, or with any methods whatsoever. Depending on our basic assumptions, the interpretation of an experiment can go either way. Poincaré was well aware long before Popper made falsification a corner stone in his characterization of the scientific method, of the pitfalls of falsification, and how hidden assumptions threaten to undercut the most sincere of such attempts.

After his initial analysis of the concepts of space and time, he for the sake of ease and convenience, allows them to be conventionally understood not to unduly complicate the discussion to follow on mechanics and physics. In fact physics to Poincaré is more or less synonymous to mechancis, after all he is writing at the very turn of the century, before the revolutions brought about on one hand Einstein and on the other hand Quantum Mechanics, which after all do not significantly alter our 'mechanical' viewpoint of the universe. To start with mechanics, you need to start with Galileo and Newton.

At least as a mathematician one is struck by the division between geometry and mechanics, the former is a part of mathematics, the latter is a part of physics. The first of the mind based on pure deductive thought, the second of the world and ultimately based on experiment. Remarkable so to speak because both are ultimately based on our sensory experiences. How come? Maybe it is but a late classification, and supposedly in Britain the tradition is still to consider classical mechanics as a branch of pure mathematics[5] To Newton and his contemporaries maybe there was no real difference. Newton based himself on Euclid and his Principia is pervaded by a geometric spirit and the enunciations of additional axioms such as his three laws. Mechanics simply being dynamic geometry with one more dimension. And surely to most outsiders, there is little if any difference between mathematics and physics, to the vulgar there is the same case of numerical problems, and

---

having it exposed to changing visual feedbacks from a computer screen, as a result of various combinations of key-strokes

[3] Still this is something most animals seem to master, and just as Chomsky speculates about an innate ability for language, this time restricted to men, that makes learning so fast, accurate and effortless: we may postulate a plethora of such innate skills, making learning more or less automatic, sight, obviously related to our space sense, but going beyond it, is another one such, that Chomsky brings up as an exemplar for the language structure.

[4] pace Gauss apocryphal measurements of angles.

[5] Poincaré notes that Maxwell is very difficult to read for a Frenchman, who expects clear definitions, lucid arguments, logical disposition and definite conclusions (as in a mathematical text?), which to some extent seems to argue against such a tradition.

to the somewhat more sophisticated, physics is about differential equations, and just as mathematics proves a handmaiden to physics, physical ideas and intuition can be quite helpful in solving mathematical problems. A two-way traffic that has become even more pronunced in recent decades. Maybe the axioms of Newton are less self-evident than the axioms of Euclid. Some of them goes against our quotodian expectations, such as that a body on which no forces are at work, continues in a straight line with a constant velocity. Clearly to enunciate such laws takes a higher kind of intuition than is required to form a workable spatial awareness. Abstractedly this observation of Galileo, certainly not the result of experimentation, or even reduible to such, is about the invariance of frames of reference moving at uniform velocities with respect to each other. This is clearly the way to think of it, reminiscent of Poincarés idea that the group of solid motions is clearly what is basic to geometry (and our psychological appreciation of the same.). Now Poincaré takes a close look at the celebrated Newtons laws and concludes that they have no physical content whatsoever, they are just definitions. What is meant by $F = ma$? What is force what is mass? [6] One way of weighing is of course to use a spring, but this will only make sense if we assume the second law of Newton, as to the case of action and reaction. And what is force? we have a muscular intuition about it, but this of course does make little sense on a celestial scale. Are we not reduced to, as was Kirchoff, to define force as the product of mass and acceleration? Thus the laws are but tautologies, having no more connection to physics than the words of our language have to external objects out there. Their value lies in their usefulness, that they allow us to organize our knowledge in elegant and graspable ways. This after all is no mean feat, so although we cannot connect them directly to the physical world, the way they structure our minds and our thinking on the matter of physics, turns out to be very useful - so far, So far! Poincaré is well aware (long before Popper, and along with many of his predecssors), of the tentative nature of science. Similarly with the constancy of energy and the minimum of action, the cornerstones of Hamiltons formalization of mechanics, in which (in the terminology of Poincaré) two entities $U, T$ (potential energy and kinetic) are defined as functions of certain variables, and the crucial entities to consider are the energy $U + T$ and the action $U - T$. What energy and action really are are less important than how they are defined and that they are constant and minimal respectively. In fact ultimately the invariance of energy does not really mean anything by itself, just that something remains constant. On a certain abstracted level this is a tautology similar to the principle of natural selection, in which what survives survives. But some tautologies are more useful than others[7], and as to the constancy of energy, when it is not satisfied we look for hidden energy[8]. This turns out to be a very useful stratagem. The key to Maxwell is his identifying the analogies of $U$ and $T$ in electro-dynamics, and then making it into a mechanical theory. The rest is

[6] the tricky question of measuring lengths and times is of course already disposed of, by assuming a conventional unproblematic notion of space and time, as not to unecessarily obfuscate the basic difficulties.

[7] This is clearly analogous to the case of imporant theormes in mathematics being eventually turned into definitions.

[8] Just as equations of movements should only depend on derivatives up to second order. If higher derivatives occur, Poincaré notes, we can always introduce invisible particles, corresponding to the trick of introducing new variables in a system of ODE's to lower the degrees.

history. Maxwells equations, I would say, was not a mere mathematical model, more could get out of it than was put into it. Lorentz recognized its invariance group and that was the first step towards special relativity, in which Poincaré also would play an important role[9].

Probability is a big thing in physics, and maybe even bigger in science in general. The idea being that the truths of science are just approximate, their conclusions just probable, and hence tentative. While in mathematics, no matter how much evidence amassed, a thing is not considered proven true unless there is a watertight deductive argument. In science this is in general not an option, we have to accept things, be it tentatively, on the basis of insufficient evidence, but as Poincaré notes on sufficient reason. If nothing really contradicts a favourite hypothesis of ours we may as well accept it, at least for the time being. Typically a hypothesis is formed because it is congruent with a satisfying simple explanantion. At least in physics, explanations are based on simplicity, it is not that probabilities are computed and if exceeding a certain cut-off values they are considered as being close to certain (the way much research is done in clinical medicine, trying to find high correlations between unrelated phenomena, and because of such numerology conclude a link). The way probabilities are usually employed in fundemental research in physics is not by the computation of actual probabilities, save in standard error analysis, but by finding two independent verification of the same phenomena, without bothering to compute actual probabilities, as such would be not only pointless but impossible, unless some, maybe unwarranted assumptions are made. In fact any calculation, in particular any probability calculation, is based on some a priori assumptions. In the case of probability on an a priori probability model i.e. distribution (and even if we make some a posteriori estimates, we put a measure on all the possible distributions we are considering among which to choose.) Confidence intervals are one thing in purely statistical research, hardly relevant when it comes to building up theoretical models. Probability from a purely scientific perspective is about saying something relevant about the unknown. As an example Poincaré takes the distribution of the longitudes of asteroids. No matter what initial distribution of longitudes and velocities, in the long run the distribution will be uniform[10]. This being a

---

[9] The role of Lorentz and Poincaré in special relativity is a controversial issue. It is often claimed that they never did get their proper dues. Armand Borel looked into it (Henri Poinacré and special relativity; L'Ensign. Math. $2^e$ série, tome 45, fasc. 3-4, 1999) and concluded that Einstein deserves the unique credit after all. If I understand his argument right, he concludes that the formalist attitude of Poincaré prevented him from drawing the (meta?)physical conclusion about the invariance of the speed of light, which is the cornerstone of relativity theory, and which suggested to Einstein the far less sexy name of 'invariant theory'.

[10] Poincaré does not get into any details, and writes no formulas, only drops a few key-words enabling the professional to reconstruct. In the case of the distribution of asteroidal longitudes, we consider a continuous distribution in the (infinite) cylinder of the longitudes $b$ on a circle, and velocities $a$ on a line. Given an initial distribution $\phi(a,b)$ we consider a small interval around $b_0$ and the integral of the area $(a,b) : |b + aT - (N + b_0)| < \epsilon$ for $T$ large. That is, suitably normalized we have $\lim_{T\to\infty} \sum_{N=-\infty}^{\infty} \int_0^1 (\int_{(N+b_0-b-\epsilon)/T}^{(N+1+b_0-b+\epsilon)T} \phi(a,b)da)db = 2\epsilon \int\int \phi(a,b)dadb$ regardless of $\phi$. Of course depending on the function $\phi$ the value $T$ may be forced to be very large.

typical application of probability for Poincaré, deriving from what is basically unknown - the initial distribution, except some continuity assumptions, which although meaningless for a discrete set. in a very elegant way solves the problem of disposing with exceptional conditions (of measure zero) without having to engage in some pedantic analysis.

The last chapter is the most technical and to the philosophically attuned maybe the least satisfactory. It contains a systematic account of Ampere and electro-dynamics, which to the uninitiated is not too easy to follow.

The remarkable thing about the essay is that in spite of being written, as noted above, before the great revolutions in physics of the early 20th century, it really is not dated. True there are repeated references to the Ether, which Poincaré predicts soon to be abandoned as an unnecessary hypothesis (which it subsequently was), and the author appears rather sceptical about the real existence of atoms, something later to be 'proved' by Einstein. Poincaré is careful not to make any prophecies as to the future of physics, well aware that such could well be made to look ridiculous already during the interval between the book being ready for the press and meeting the reading public.

September 7, 2009 **Ulf Persson:** *Prof.em, Chalmers U.of Tech., Göteborg Sweden* ulfp@chalmers.se