Space,Time and Gravitation

An Outline of the General Relativity Theory

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Eddington led one of the famous expeditions during the total eclipse of 1919 to test the predictions of Einstein as to the refraction of light from distant stars passing close to the sun. Eddington went to Principie in Africa, the other better equipped went to Brazil. The total eclipse was a good one, because the sun was in the vicinity of many relatively bright stars, had that not been the case, it would have been far more difficult to make the appropriate measurements. The confirmation of Einstein created a public sensation and Einstein overnight became a household name and has remained one almost a century later. However, the story is much more complicated than one would have believed, experimental science is different from theoretical, messy and pedantic, pedestrian in spirit, unforgiving as to detail, and ultimately diffuse. For one thing that light was refracted by gravitation was also inherent in Newtons theory, while the predictions of that was only half of those predicted by Einstein. Thus it was not just a matter of predicting apparent displacement, it was also one of predicting its rather precise amount. The evidence took some time in being collected and interpreted and at first it came across as rather contradictory. At the end, however, there was a clear, but of curse not definitive corroboration, such things are never to be had according to Popper. The rather lengthy chapter is a diversion from the unrelenting theoretical account given by Eddington, one of the first in the Anglo-Saxon world to fully appreciate and absorb the new revolutionary ideas from the continent.

The book is written in a philosophical vein, which is of course very appropriate, because at the heart of the revolution was a revolution in philosophy, the philosophy of space, which had been also at the heart of Newton's celestial revolution more than two hundred years earlier, and going all the way back to Euclid. The question is what is meant by geometry, how can we compare one length with another, is there any absolute sense of rigidity? Or is it something we merely define by convention? Eddington starts out with a classical dialogue, or rather trialogue, involving three people, the die-hard realist physicist, the formal absent-mattered mathematician[1] and the wise relativist, the latter being of course a mouthpiece for Eddington himself. The purpose of the whole exercise is to get rid of Euclidean geometry, this is no problem for the mathematician of course, who plays a marginal role in the interchange, having already made peace with non-Euclidean geometry a long time ago, but a hurdle for the more down-to-earth physicist.

The name relativity theory is of course very catchy, and it fitted well into the 'Zeitgeist' at the time, while Einstein himself had preferred the title 'invariant theory', because after all, the pivotal point, is the invariance of the speed of light independent of frames

[1] In the sense of Russell, knowing nothing of which he speaks, and caring less of whether it is true or not and whether it pertains to anything real in the world. A logicians caricature.

of reference. That things far apart seem small to us is something we learn very early on and incorporate in our navigation. We find it not at all paradoxical that we will be seen by distant people as small as they seem to us, because we can make allowances for a 3-dimensional world in which our 2-dimensional fields of vision are seen as just accidental points of view contingent upon an observer. As far as visual image is concerned, we simply integrate the various 2-dimensional subjective images into an objective 3-dimensional world[2], which we only know by inference, but which we nevertheless think of as real and palpable[3]. Relativity theory is in principle not different, only that time is added, and time too becomes relative, not just the appearance of space. Mathematically there is no difference at all as to general principles, but psychologically there is a large leap, as we think of time as being so essentially different than space. The beautiful mathematical synthesis of 4-dimensional space-time was presented by Minkowski. And that leads to the question of whether you can really appreciate relativity theory without mathematics? The answer is no. Eddington does his best but to entirely divorce the two is impossible, and there are some rather advanced mathematical reasoning in his non-mathematical text, even if he tries to eschew formulas, and out of desperation he does add in an appendix some 16 mathematical notes, not extensive ones, but only suggestions, in order to bridge the gap with a more mathematical treatment.

Mathematicians and theoretical physicists are Platonists, whether they accept it or not. Far from being a celebration of relativity, Einsteins synthesis was a celebration of an unchanging, absolute reality behind the confusion of our immediate senses. If ever a literal Platonic quest. Special relativity sets the stage, but relativity only comes to its own, when general relativity is considered. Why confine yourself to frames in uniform motions with respect to each other, why not considering arbitrary frames? When it comes to frames with uniform motions, you can never tell which one is moving and which is at rest, in fact you can chose as convention any at rest. It is different with accelerated frames, such as rotating. As Newton pointed out, the Earth is really rotating in a real absolute sense, it is not that we can arbitrarily set the Earth at rest and make the celestial firmament go around. The rotating earth has physical consequences which we can observe, such as the bulging at the equator, the gyroscopic effects later displayed by Focault and his pendelum or the coriolis forces deviating winds and oceanic streams. If we set the earth at rest, as we automatically do as terrestrial inhabitants, we experience phantom inertial forces. The major idea underpinning the general theory of relativity is that this distinction between phantom forces and real ones such as gravitation does not exist. Is a phantom in fact. There is in Newtonian mechanics a distinction between passive mass and active mass, the former known as inertia, the reluctance to be moved, the latter known as weight or heavy mass, actively generating attraction. By coincidence heavy and inert

[2] Mathematically this is simple as anyone who has tried his or her hands at 3-dimensional computer graphics knows. We simply set up formal arrays of three variables and then use elementary linear algebra to effect arbitrary two-dimensional projections.

[3] One wonders whether our biologically evolved sense of distance is set aside when we are in an airplane. Can we not imagine that the ground is just ten meters below at most, and what we see is just a miniature model of the world, where men are ants and cars are toys? We know of course that this is not true, but that has little bearing on our local, physical knowledge.

mass are proportional, i.e. the same, but by identifying them it becomes no longer an issue. In picturesque language, the 4-dimensional space-time is curved. Mass changes the geometry, which imposes the way mass can move. Thus the 3-dimensional dynamic world of Newton becomes a static geometrical 4-dimensional one of Einstein. As Popper remarked, he had achieved a Parmenidean synthesis. The worldlines of particles follow geodetic curves in the manifold, those being curves that locally maximize the relevant distance[4]. The ideas may be simple, but it leads to mathematical difficulties which were beyond those of Einstein to master from scratch. Luckily the mathematicians of the 19th century had provided what to Einstein would serve as tools. Thus he was able to set up the Einstein equation which governed the behavior of the 4-dimensional space-time manifold. A theory both simple and subtle enough to provide feed for mathematical investigation still a century later. In it one can surmise some interesting things, such that acceleration being a more fundamental entity than velocity, although it is usually defined in terms of changes of velocity. General relativity explained many puzzles of the past, such as the exaggerated pace of the precession of the orbit of Mercury[5]. With the mapping of the universe and discovery of the red-shift interpreted as the receding of galaxies proportional to their distance general relativity is at the basis of cosmology, and the eventual fate of it will depend on the amount of mass present. While Eddington discusses at some detail the connection with mass and the extension of the universe (without mass there would be no need to make space!) he is as of yet ignorant of its expansion.

The staring point of relativity theory was the negative results of the experiments of Michelson-Morley in the 1880's, in which no movement of the earth with respect to the supposed ether (absolute space) was detected over the revolution around the sun, Had that experiment been undertaken in the 16th century it would of course have given undeniable support to the common-sense idea that the earth was immobile in space, but times and understanding had changed irrevocably since then. The phenomenon was explained by contraction in the direction of movement, so called Fitzgerald contraction[6] causes for which were given very convincing electro-dynamic explanations. All of those sophisticated explanations have no receded into general oblivion as a result of the paradigm shift effected by Einstein. Thus Einstein did not need to provide an explanation for Michelson-Morley, such existed satisfactorily, he was presumably propelled by a purer philosophical motivation.

In the penultimate chapters the author discusses, as provisional, the theory of Weyl, later to be known as gauge theory, incorporating electrodynamic phenomena within the framework of relativity theory. As is well known, the equations of Maxwell incorporated the symmetries of the 4-dimensional space-time, known as the Lorenz transformations.

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[4] The crucial mathematical difference from Euclid is that the quadratic form is not positive definite but indefinite, in fact with index $1, 3$. This leads to the light-cone and other typical phenomena having to to with the demarcations of causality.

[5] Why Mercury? Mercury is the closest planet to the sun, hence with the fastest orbital speed and the quickest completion of a revolution, allowing a more pronounced effect to be accumulated at a quicker pace. Furthermore, unlike the orbits of Venus and the Earth its orbit is pronounced elliptical, meaning that it has high eccentricity and hence a well-defined major axis, the position of which it is much easier to pinpoint with required accuracy.

[6] Proportional to $\sqrt{1 - v^2/c^2}$

This included the phenomenon of holonomy, could it be that a rigid length unit when moved around and returned back to its point of origin had changed length? Recall that in this case we can make a direct comparison.

He ends on a philosophical note, as to what really constitutes reality. Has the 4-dimensional space reality, or is it just a convenient imagination of our brains, useful merely for computational purposes, the so called instrumental view? But of course the same question can be asked about our 3-dimensional world, and then it appears somewhat perverse. Are there laws out there in nature, which are so strange that the human mind is incapable of conceiving them let alone formulating them[7].

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[7] Often in science and mathematics, formulation comes before conception and understanding, a triumph of formalism.