

An Introduction to Mathematics

A.N.Whitehead

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Any reader that expects to get a philosophical explanation of mathematics is bound to be disappointed. The purpose of the book is more modest, without being entirely devoid of pretension, and essentially didactic. Whitehead wants to tell the beginning student of mathematics of what it is all about saving him or her from being drowned in technicalities. There might be some merit to such an ambition, after all much of the mathematical education is one of drudgery in which many get lost. Surely it could be done in some more efficient way? Or can it? Whitehead remarks that in the history of mathematics, explanations of what is really going on, have always been provided retrospectively in the light of some new insight. Could it be differently with an individual? Maybe each person has to go through a painful process, after all there is famously no royal way to geometry. Some students instinctively catch on quickly, they sense what is at stake, and why certain things are being done. They are what one think of as mathematically gifted, with a right temperament for abstraction, its purposes, possibilities as well as limitations. But even those prosper by returning to well-trodden grounds. Facts of mathematics, even if encountered in isolation, do not make sense unless in a context. Mathematical education is a matter of putting some things in repeatedly different contexts and only through this process will a better and better understanding arise. Whitehead tries a shortcut, briefly going through the elementary material, such as numbers, vectors, complex numbers, geometry, conic sections, trigonometry, periodicity and limits and calculus. In doing so he is often rather technical and long-winded and seldom elegant, and indeed a student may either know the material already (In which case he will be less motivated to peruse it) or be hopelessly in the dark having no previous exposure to it. In order to really profit from a survey, you need to have some intimate acquaintance with it to start out with. It is very doubtful that this book could have been helpful in alleviating so called Math anxiety, a condition present a hundred years ago as well, although not diagnosed as such. Just because a book is old does not mean that it is a classic.

So what are the redeeming features of this book? Does it give any helpful philosophical insights? Are there any striking formulations, be it of things well-known and universally accepted. If not, should it be condemned to the flames, following the advice of a Hume? It does emphasize the importance of pure research, but also that some important pure ideas have arisen out of practical needs, such as negative numbers. Also that great discoveries usually are in the air, and the releasing insight might be likened to the sun-rays that set off an avalanche. Small causes, great effect, only because things have been properly set up. Mathematics is about increasing the number of new ideas, not to just add routinely to an output. A subject can be studied for thousands of years without there being any progress, until a new idea revolutionizes it. Geometry being such an example, re-invigorated in the 17th century by Descartes and Fermat. The point of symbolic manipulation in mathematics is the relieve the mind of thought. Manipulations

should be done automatically and mechanically without inquiring at each step of what is being done. This is the great advantage of mathematical terminology, freeing the mind for more important things. Numbers are more fundamental and more abstract than geometry, having unlike geometry, no preferred intuitive manifestation. We can count the number of imaginary things, like archangels, without being able to position them in space. Finally time is based on conventions, regular phenomena which we assume more regular than anything else. Eventually we adopt a definition of time compatible with the most sophisticated theory we can conceive of. That the days are getting longer is something we could not really discern from empirical scrutiny, only something we derive from Newtonian mechanics. To assume that the days are of equal length, would mean to reject Newton. The inconvenience of so doing would be too high a prize to pay.

Whitehead says that philosophers, who know little mathematics or have come to mathematics late, cannot contribute to the philosophy of mathematics. What they tend to come up with is either false or trivial. Sometimes one uncharitably wonders whether that harsh judgment also can be applied to the author himself.

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