Sociological Methodology

Comment: Are Marginal Models Needed?
Nanny Wermuth

The online version of this article can be found at:

Published by:
⑤SAGE

On behalf of:

AMERICAN SOCIOLOGICAL ASSOCIATION
American Sociological Association

Additional services and information for *Sociological Methodology* can be found at:

Email Alerts: http://smx.sagepub.com/cgi/alerts

Subscriptions: http://smx.sagepub.com/subscriptions

Reprints: http://www.sagepub.com/journalsReprints.nav

Permissions: http://www.sagepub.com/journalsPermissions.nav

>> Version of Record - Sep 5, 2013

What is This?

COMMENT: ARE MARGINAL MODELS NEEDED?

Nanny Wermuth*

*Chalmers University of Technology,
Corresponding Author: Nanny Wermuth, wermuth@chalmers.se

It is a pleasure to comment on three papers, where each provides substantial new work and each mentions marginal models in the title.

The paper by Kuijpers, Van der Ark, and Croon (this volume, p. 42–69) presents developments in the construction of scales for categorical variables. A sequence of papers has been published in recent years in the psychometric literature on properties of this type of nonparametric scale construction for binary and polytomous items. The items here are the observed variables, each having the same number of categories or levels.

A first program version for analyses was integrated several years ago into the computing environment R and updated recently by Van der Ark (2012). As a result, a well-written, thoughtful use of this type of analysis is, for instance, now available for health researchers, as discussed in Stochl, Jones, and Croudace (2012).

The earliest discussions of these scales appear to be by Loevinger (1947) and the new result in the current paper is the derivation of standard errors for functions of simple correlation coefficients among the items. For these, Mokken (1971) had formulated several rules of thumb as guidelines to judge the quality of a scale.

The assumptions for a Loevinger-Mokken scale build on many years of research in the area and may be summarized as

- a single unobserved (called a latent) variable, which fully explains the associations observed among the items;
- the dependence of each item on the latent variable is positive and similar, in the sense of being nondecreasing with the levels of each item and changing from item to item so little that these dependence curves do not intersect.

These assumptions are taken as justification to use a simple sum of the items as an appropriate summary score, where the score is to measure an ability, an attitude, a medical status, or another property of an individual. But checks for the above first assumption appear to be currently not included in reported analyses—that is, for conditional independence of the items given the latent variable, also called "local independence."

The outcome of the analysis for a single Loevinger-Mokken scale is analogous to a Gaussian factor analysis result for an equal correlation matrix with positive correlations, say ρ. For m items, this type of matrix has one large eigenvalue λ_{max} and $m-1$ equal eigenvalues of size $1-\rho$. The eigenvector of λ_{max} justifies the simple sum score since it is a multiple of the unit vector.

The generating process for factor analysis with a single factor or for a unidimensional scale of categorical variables can be represented by the same type of a simple directed graph, as discussed in Cox and Wermuth (2002) and shown in Figure 1. Recent results for recursive sequences of single or joint response regressions can be used to derive common features of generating processes that differ in the type of variables involved but have the same generating graph.

These results concern the construction of summary graphs after ignoring some variables and/or selecting subpopulations (Wermuth 2011; Sadeghi and Marchetti 2012), on Markov equivalence of regression graphs (Wermuth and Sadeghi 2012), and on traceable regressions—that is, on conditions under which pathways of dependence can be traced using only the graph (Wermuth 2012).

We illustrate this here with five items and a single latent variable L. Figure 1 (a) shows a directed star graph in six nodes with each item j
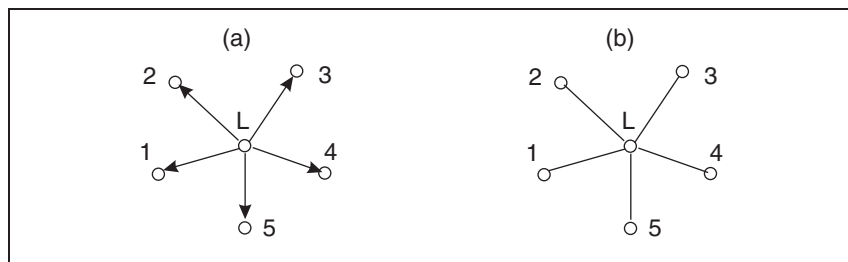

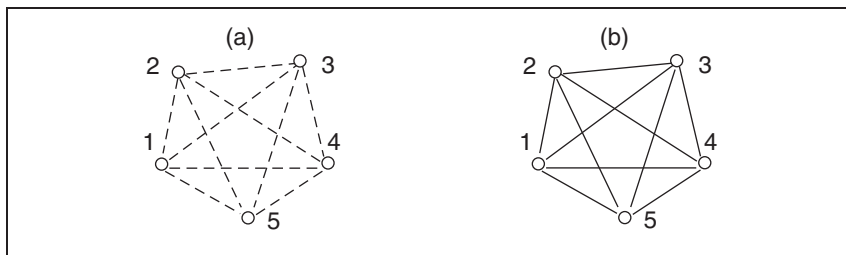**Figure 1.** A directed star graph (a) and an undirected star graph (b).

Figure 2. Induced, complete graphs for the items, with the covariance graph in 2(a) being induced by 1(a) and the concentration graph in 2(b) induced by 1(b).

depending on L. Figure 1 (b) shows the corresponding Markov equivalent undirected graph with item j depending on L given the remaining items.

The factorization of the generated family of densities in $N = \{1,2,\ldots,5,L\}$ is, expressed in a compact node notation,

$$f_N = f_{1|L}, \ldots, f_{5|L}f_L. \tag{1}$$

Marginalizing over the single common inner node L gives the complete covariance graph in Figure 2 (a) for Figure 1 (a) and a complete concentration graph in Figure 2 (b) for Figure 1 (b).

The relevant interpretation is here for subgraphs. A generating directed star graph implies for each set of distinct items h, j, k and for positive dependences of each j on L:

- h is marginally dependent on j,
- h is dependent on j given k,
- the type of dependence of j on h is qualitatively similar for each level of k.

What is being suggested is that these necessary conditions for the existence of a simple sum score could be routinely checked. It may also be possible to derive stronger conditions. In addition, it seems that the above, fully internal method of the scale construction may need some sort of external supplementation. This may involve checking the scale for different populations or its predictive ability for relevant responses.

In the paper by Bergsma, Croon, and Hagenaars (this volume, p. 1–41), several new applications of marginal models are presented. Again, I think that it is an important step forward that the results of the data analyses can be reproduced using an R package; see Bergsma and van der Ark (2012). I fully agree that marginal models will be helpful for several nonstandard situations and that it improves understanding to see such case studies. These should include cases in which computational difficulties arise, such as those mentioned by the authors.

However, in my experience, it does not happen very often—as claimed by the authors—that investigators have no "substantive interest in the nature of the dependences" in their collected data. A key example is the author's first motivating data set. It is on square contingency tables with matching ordinal rows and columns. The marginal distributions are compared at several time points without studying the actual differences that lead to the observed overall changes in the margins.

Even if standard textbook descriptions ignore changes between such time points, simple methods for concentrating on the essential changes have been derived; see Cox and Jackson (2009), where cohort comparisons regarding social class and income mobility are studied.

Differences in success probabilities, as recommended in Section 4.2 of the discussed paper, may sometimes be useful, but it is only an odds-ratio that does not depend on the margins; see Edwards (1963). There is also a more recent, careful comparison of properties of different, possible measures of association by Xie, Ma, and Geng (2008).

The paper by Németh and Rudas (this volume, p. 70–100) gives substantively important insights using a directed acyclic graph for data on the social attainment process in different types of welfare systems. In the author's first constructed set of data, two variable pairs, (M, L) and (F, L), are marginally independent. Nevertheless the pair (M, F) is jointly dependent on L due to a strong three-factor interaction.

This captures a nonstandard situation in search strategies for important regressors within linear or generalized linear models having a multivariate response variable. It is important to be alert to such situations in which higher-order interactions may coexist with marginal independences. But trying to explain models with directed acyclic graph structure in terms of marginal models is a bit like using a sledgehammer to crack a nut.

For each complete, fully directed, acyclic graph, the generated joint distribution factorizes into a recursive sequence of single response

regressions. For instance, the given ordering for the model analyzed in Section 5, $N = (I,O,E,G,F)$, implies that only *E, G, F* are possible regressors for *O*. Hence such a single-response regression model is a marginal model in the sense of involving not all variables. But, a simpler factorization such as

$$f_{O|EGF} = f_{O|EG},$$

as implied by the missing edge $O \leftarrow F$ in Figure 7 of the paper simply means that the potential regressor *F* is not needed to generate *O* when *E, G* are available as regressors.

Many analyses for recursively ordered responses have now been reported in the literature, also with ordinal or with joint responses; see for instance Wermuth and Cox (1998, 2013) and Wermuth and Sadeghi (2012). To see whether the results permit path analyses as introduced by geneticist Sewall Wright a century ago, one has to check whether the additional properties of traceable regressions are satisfied. Wright had coined the term *path analysis* for the generating processes he studied, in which every arrow in a fully directed, acyclic graph exclusively represents a strong, linear dependence.

Bergsma and Rudas (2002) defined the class of marginal models in the *Annals of Statistics* more than a decade ago. Since then, they have intensively studied further properties and applications, and they have also done so jointly with colleagues. Now, one can decide for each model in the class whether it has some nice properties or does not. But, as with almost each large model class, it contains models that are difficult to interpret.

In addition, the class may not be needed for more traditional tasks and models—for instance, to derive variances or to understand regression models for discrete variables. It would be nice if similar results, as for marginal models, became available for data that contain both categorical and quantitative variables. In my experience, this is the most frequent situation in applications.

References

Bergsma, Wicher P. and Tamás Rudas. 2002. "Marginal Models for Categorical Data." *Annals of Statistics* 30(1):140–59

Bergsma, Wicher P. and L. Andries Van der Ark. 2012. *Categorical Marginal Models. R Package cmm*, Version 0.4.

Cox, David R. and Michelle Jackson. 2009. "On Square Ordinal Contingency Tables: A Comparison of Social Class and Income Mobility for the Same Individuals." *Journal of the Royal Statistical Society,* Series A, 172:483–93.

Cox, David R. and Nanny Wermuth. 2002. "On Some Models for Multivariate Binary Variables Parallel in Complexity with the Multivariate Gaussian Distribution." *Biometrika* 89:462–69.

Edwards, Anthony W. F. 1963. "The Measure of Association in a Table." *Journal of the Royal Statistical Society,* Series A, 126:109–14.

Loevinger, Jane. 1947. "A Systematic Approach to the Construction and Evaluation of Tests of Ability." *Psychological Monographs* 61:4.

Mokken, Robert J. 1971. *A Theory and Procedure of Scale Analysis.* The Hague, Netherlands: Mouton.

Sadeghi, Kayvan and Giovanni M. Marchetti. 2012. "Graphical Markov Models with Mixed Graphs in R." *The R Journal* 4:65–73.

Stochl, Jan, Peter B. Jones, and Tim J. Croudace. 2012. "Mokken Scale Analysis of Mental Health and Well-being Questionnaire Item Responses: A Non-parametric IRT Method in Empirical Research for Applied Health Researchers." *BMC Medical Research Methodology* 12(74):1–17.

Van der Ark, L. Andries. 2012. "New Developments in Mokken Scale Analysis in R." *Journal of Statistical Software* 48:1–27.

Wermuth, Nanny. 2011. "Probability Models with Summary Graph Structure." *Bernoulli* 17:845–79.

Wermuth, Nanny. 2012. "Traceable Regressions." *International Statistical Review* 80:415–38.

Wermuth, Nanny and David R. Cox. 1998. "On the Application of Conditional Independence to Ordinal Data." *International Statistical Review* 66:181–99.

Wermuth, Nanny and David R. Cox. 2013. "Concepts and a case Study for a Flexible Subclass of Graphical Markov Models." Pp. 331–50 in *Robustness and Complex Data Structures* (Festschrift in honor of Ursula Gather), edited by Claudia Becker, Roland Fried, and Sonja Kuhnt. Heidelberg, Germany: Springer.

Wermuth, Nanny and Kayvan Sadeghi. 2012. "Sequences of Regressions and Their Independences" (with discussion). *TEST* 21:215–79.

Xie, Xianchao, Zongming Ma, and Zhi Geng. 2008. "Some Association Measures and Their Collapsibility." *Statistica Sinica* 18:1165–83.

Author Biography

Nanny Wermuth is professor emerita of Statistics at the Department of Mathematical Sciences, Chalmers University of Technology, and guest professor at the Department of Medical Psychology, Johannes-Gutenberg University of Mainz, with a main research interest in graphical Markov models and their applications in the life sciences and in the natural sciences.