Svar LMA033/515a BI/KI 13/1-2012

1a $\dfrac{x-y}{2x}$ 1b $\dfrac{1}{2(x-2)}$

2a $\dfrac{x^2}{2x-3} = x+2 \iff x^2 = 2x^2 + x - 6$

$\iff x = -\frac{1}{2} \pm \sqrt{\frac{1}{4}+6} = \dfrac{-1 \pm 5}{2}$

$x_1 = -3 \quad x_2 = 2$

2b $(1-2x)^2 = 3-2x \iff 1-4x+4x^2 = 3-2x$

$\iff 4x^2 - 2x - 2 = 0 \iff x = +\frac{1}{4} \pm \sqrt{\frac{1}{16}+\frac{1}{2}}$

$x_1 = 1 \quad x_2 = -\frac{1}{2}$ svar: $x = -\frac{1}{2}$

3a

$-4 \quad -2 \quad 3 \quad x$

$- \quad + \quad - \quad +$

$x < -4$ el. $-2 < x < 3$

3b $\dfrac{x}{x-1} > \dfrac{1}{x+3} \iff \dfrac{x(x+3)-(x-1)}{(x-1)(x+3)} > 0$

$\iff \dfrac{x^2+2x+1}{(x-1)(x+3)} > 0$

$-3 \quad -1 \quad 1$

$+ \quad - \quad - \quad +$

$x < -3$ el. $x > 1$

4

$-1/2 \quad 2 \quad x$

I II III

$x \le -1/2$

I: $x(2-x) = -2x$

$x^2 - 4x = 0$

$x = 0,\ x = 4 \notin$ I

$-1/2 < x \le 2$

II: $x(2-x) = 2x+2$

$-x^2 = 2$ ingen lösn

III ($x > 2$): $x^2 - 2x = 2x+2$

$x = 2 \pm \sqrt{6}$ $\boxed{x = 2+\sqrt{6}}$

5a $\ln(2(x+1)) = \ln x^2$

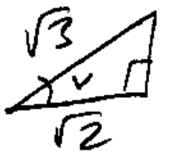
$x^2 - 2x - 2 = 0 \quad x = 1 \pm \sqrt{3}$

$x = 1 + \sqrt{3} \quad ((\ln(1-\sqrt{3})$ ej def$)$

5b $t = 2^x \quad t^2 + \frac{1}{2}t - \frac{1}{2} = 0$

$t = -\frac{1}{4} \pm \sqrt{\frac{1}{16} + \frac{1}{2}} = \frac{-1 \pm 3}{4} \quad t_1 = -1 \quad t_2 = \frac{1}{2}$

$2^x = -1$ saknar lösn $\quad 2^x = \frac{1}{2} = 2^{-1} \quad x = -1$

6a  $\sin v = 1/\sqrt{3}$

6b $\sin(2v) = \sin(v + \frac{\pi}{3})$ 

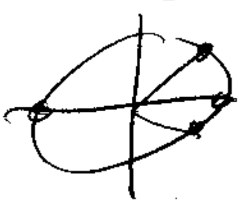
$2v = v + \frac{\pi}{3} + n\,2\pi \qquad 2v = \pi - (v + \frac{\pi}{3}) + n\,2\pi$

$\underline{v = \frac{\pi}{3} + n\,2\pi} \qquad 3v = \frac{2\pi}{3} + n\,2\pi$

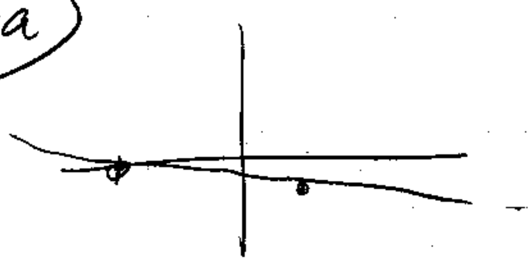
$\underline{v = \frac{2\pi}{9} + n\frac{2\pi}{3}}$

6c $2\sin v = \sqrt{3}\,\frac{\sin v}{\cos v} \iff \sin v = 0$

eller $\cos v = \frac{\sqrt{3}}{2} \iff v = n\pi$ eller



$v = \pm\frac{\pi}{6} + n\,2\pi$

7a 

$\frac{y-0}{x-(-5)} = \frac{0-(-1)}{-5-2} = -\frac{1}{7}$

$y = -\frac{x}{7} - \frac{5}{7}$

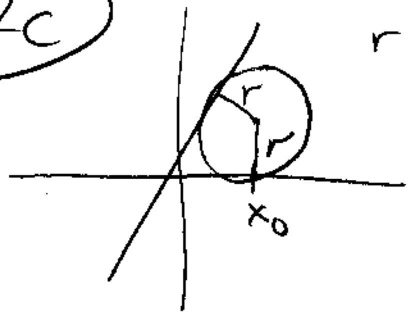
(7b) $(x+2)^2 - 4 + 2\left(\left(y-\frac{3}{4}\right)^2 - \frac{9}{16}\right) = -5$

$(x+2)^2 + 2\left(y-\frac{3}{4}\right)^2 = \frac{9}{8} - 1 = \frac{1}{8}$

$$\frac{(x+2)^2}{\frac{1}{8}} + \frac{\left(y-\frac{3}{4}\right)^2}{\frac{1}{16}} = 1$$

medelpkt $(-2, 3/4)$ halvaxl. $\frac{\sqrt{2}}{4}, \frac{1}{4}$

(7c)



$r=1$ ekv: $(x-x_0)^2 + (y-1)^2 = 1^2$

$y=2x$: $(x-x_0)^2 + (2x-1)^2 = 1$

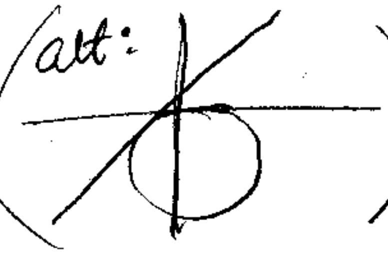
$5x^2 - (2x_0+4)x + x_0^2 = 0$

$$x = \frac{x_0+2}{5} \pm \sqrt{\underbrace{\frac{(x_0+2)^2}{25} - \frac{x_0^2}{5}}_{=0,\ \text{ty}}}$$

$(x_0+2)^2 = 5x_0^2$

$$x_0 = \frac{2}{\sqrt{5}-1} = \frac{\sqrt{5}+1}{2}$$

(alt:



)

(8a) $f'(x) = D\left(-2x^{5/2} + x^4 - \frac{3}{2}x + 1\right) = -5x^{3/2} + 4x^3 - \frac{3}{2}$

$t = x^{3/2}$: $4t^2 - 5t - \frac{3}{2} = 0$ $\quad t = \frac{5}{8} \pm \sqrt{\frac{25}{8^2} + \frac{3}{8}}$

$= \frac{5}{8} \pm \frac{7}{8}$ $\quad t = \frac{12}{8} = \frac{3}{2} = x^{3/2}$ $\quad x = \left(\frac{3}{2}\right)^{2/3}$

(8b) $f'(1) = -\frac{5}{2}$ $\quad f(1) = -\frac{3}{2}$

$y + \frac{3}{2} = -\frac{5}{2}(x-1)$ $\quad y = -\frac{5x}{2} + 1$