# Svar Matte intro/A BI/KI

(kortfattade, EJ fullst. lösn) 20/8 -2008

1a) $x^{-1/3}y^{1/2} = \dfrac{\sqrt{y}}{\sqrt[3]{x}}$

1b) $\dfrac{x+y}{(x^2+y^2)(x-y)}$

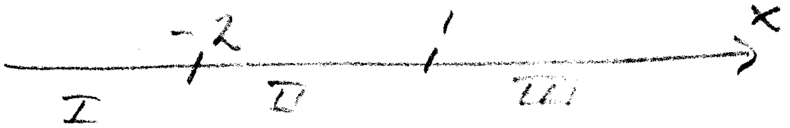
1c) $\dfrac{2(x+1)x-4(x-2)-12}{(x-2)^2(x+1)} = \dfrac{2(x^2-x-2)}{(x-2)^2(x+1)} = \dfrac{2(x-2)(x+1)}{(x-2)^2(x+1)}$

$= \dfrac{2}{x-2}$

2a) $4(5-x) = (x-3)^2 \iff x^2-2x-11=0$

$x = 1 \pm \sqrt{12} = 1 \pm 2\sqrt{3}$ men $1+\sqrt{12} > 3$

2b)



I : $3+x+2 = 2(1-x)+1 \iff x = -\frac{4}{3} \notin$ I

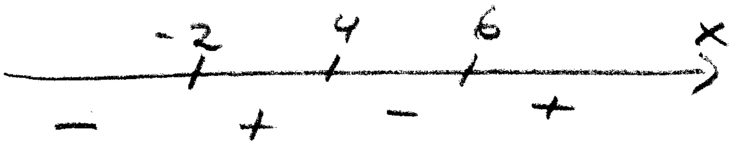
II : $5-x-2 = 2(1-x)+1 \iff x = 0 \in$ II

III : $5-x-2 = 2(x-1)+1 \iff x = \frac{4}{3} \in$ III

3a) $(x-5)^2 \geq x+1 \iff x^2-11x+24 \geq 0$

$\iff x \leq 3$ el. $\underline{x \geq 8}$ men $\sqrt{\phantom{x}} \geq 0 \Rightarrow x \geq 5$

3b) $\dfrac{(x+3)(x-4)-3x}{x-4} \geq 0 \iff \dfrac{(x+2)(x-6)}{x-4} \geq 0$



$-2 \leq x < 4$ el. $x \geq 6$

4) $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 12$

prøvn $\Rightarrow$ $x = -3$

$$
\begin{array}{r}
x^2 - 2x - 4 \\
x+3 \,\big)\, \overline{x^3 + x^2 - 10x - 12} \\
-(x^2 + 3x^2) \\
\hline
-2x^2 - 10x - 12 \\
-(-2x^2 - 6x) \\
\hline
-4x - 12 \\
-(-4x - 12) \\
\hline
0
\end{array}
$$

$x^2 - 2x - 4 = 0$

$\Leftrightarrow$

$x = 1 + \sqrt{5}$

fakt.

$(x+3)(x-1+\sqrt{5})(x-1-\sqrt{5})$

5a) $\ln 6x^2 = \ln \frac{5x}{6^2x^2} \Leftrightarrow 6^3x^4 = 5x, x \neq 0$

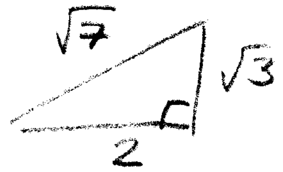
$\Leftrightarrow x = \frac{\sqrt[3]{5}}{6}$

5b) $2 = A(e^0 + e^{-0}) \Leftrightarrow A = 1$

$3 = e^{2b} + e^{-2b}$ $\quad u = e^{2b}$ $\quad 3 = u + \frac{1}{u}$

$u^2 - 3u + 1 = 0$ $\quad u = \frac{3}{2} \pm \sqrt{\frac{9}{4} - 1} = \frac{3 \pm \sqrt{5}}{2}$

$b = \frac{1}{2} \ln \frac{3 \pm \sqrt{5}}{2} = \pm \frac{1}{2} \ln \frac{3 + \sqrt{5}}{2}$

6a) 


$\sin v = \sqrt{\frac{3}{7}}$

6b) $\cos\left(\frac{\pi}{2} - 2v\right) = \cos(2v)$ $\quad \frac{\pi}{2} - 2v = \pm 2v + n2\pi$

$v = \frac{\pi}{8} - n\frac{\pi}{2}$ $\quad$ (el. $\tan(2v) = 1$)

6c) $\sin v = 2\sqrt{3}\,(1-\sin^2 v)$

$\sin v = -\frac{1}{4\sqrt{3}} \pm \sqrt{\frac{1}{48}+1} = \frac{-1 \pm 7}{4\sqrt{3}}$

men $-\frac{8}{4\sqrt{3}} < -1 \Rightarrow \sin v = \frac{\sqrt{3}}{2}$

$v = \frac{\pi}{3} + n2\pi$ el. $v = \frac{2\pi}{3} + n2\pi$

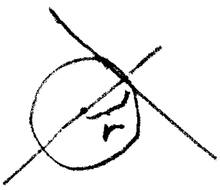
7a) $y - 1 = \frac{1-(-2)}{-2-3}(x+2) \Leftrightarrow y = -\frac{3x}{5} - \frac{1}{5}$

7b) $4\left(\left(x+\frac{1}{8}\right)^2 - \frac{1}{8^2}\right) + 2\left(\left(y-\frac{1}{2}\right)^2 - \frac{1}{2^2}\right) = 1$

$4\left(x+\frac{1}{8}\right)^2 + 2\left(y-\frac{1}{2}\right)^2 = \frac{25}{16}$

$\frac{\left(x+\frac{1}{8}\right)^2}{\frac{1}{4}\cdot\frac{25}{16}} + \frac{\left(y-\frac{1}{2}\right)^2}{\frac{1}{2}\cdot\frac{25}{16}} = 1$ centr $\left(-\frac{1}{8}, \frac{1}{2}\right)$

halvax $\frac{5}{8}, \frac{5}{4\sqrt{2}}$

7c) 


$\begin{cases} y = 2x+3 \\ y = -\frac{1}{2}(x-1)+1 \end{cases} \Leftrightarrow \begin{cases} \frac{5x}{2} = -\frac{3}{2} \\ y = 2x+3 \end{cases}$

$r = \left(\left(-\frac{3}{5}-1\right)^2 + \left(\frac{9}{5}-1\right)^2\right)^{1/2} = \frac{4}{5}\sqrt{5}$

8a) $f(x) = x - 4x^{-1/2} + 4x^{-1}$ $f'(x) = 1 + 2x^{-3/2} - 4x^{-2}$

$f(4) = 3$ $f'(4) = 1$

tgt: $y - 3 = 1\cdot(x-4)$ $y = x - 1$

norm: $y - 3 = -(x-4)$ $y = 7 - x$

8b) $f''(x) = -3x^{-5/2} + 8x^{-3} = 0 \Leftrightarrow 8x^{-3} = 3x^{-5/2}$

$\Leftrightarrow \frac{8}{3} = x^{1/2} \Leftrightarrow x = \frac{64}{9}$