Svar Matte Intro/A/ 17/8 - 2011

1a) $\dfrac{x^2+2xy-3y^2}{(x+2y)(x-y)} = \dfrac{(x+y)^2-4y^2}{(x+2y)(x-y)} = \dfrac{\cancel{(x-y)}(x+3y)}{(x+2y)\cancel{(x-y)}}$

b) $\dfrac{(x+2)(x-2)+3}{(x+1)(x-2)} = \dfrac{x^2-1}{(x+1)(x-2)} = \dfrac{x-1}{x-2}$

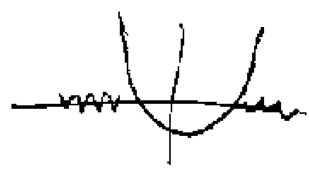
2a) $x^2-5x+4=0 \qquad x=\frac{5}{2}\pm\sqrt{\frac{25}{4}-4} = \frac{5\pm 3}{2} =$

$x=1,\ x=4 \quad \left(x=1 \Rightarrow 1-2=\sqrt{1}\right) \quad \underline{x=4}$

b) $x^2=t \Rightarrow t^2-8t+16=t+8 \Leftrightarrow t=\frac{9}{2}\pm\sqrt{\frac{81}{4}-8} \Leftrightarrow t=\frac{9\pm 7}{2}$

$t=8,\ t=1 \quad \left(t=1 \Rightarrow 1-4=\sqrt{9}\right) \quad t=8 \Leftrightarrow \underline{x=\pm\sqrt{8}}$

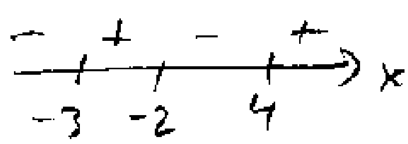
3a) $t^2-2t-\frac{5}{4}=0 \Leftrightarrow t=1\pm\sqrt{1+\frac{5}{4}} = 1\pm\frac{3}{2}$



$t<-1/2$ eller $t>5/2$

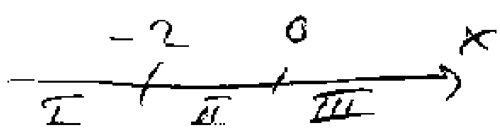
3b) $\dfrac{x^2-9-2x+1}{x+3}>0 \qquad x=1\pm\sqrt{1+8} = 1\pm 3$

$\dfrac{(x-4)(x+2)}{x+3}>0$



$\underline{-3<x<-2 \text{ el } 4<x}$

4)



I: $x^2+2x=1+x \qquad x=-\frac{1}{2}\pm\sqrt{\frac{1}{4}+1} \notin I$

II: $-x^2-2x=1+x \qquad x=-\frac{3}{2}\pm\sqrt{\frac{9}{4}-1} \qquad x=-\frac{3}{2}+\sqrt{\frac{5}{4}} \in II$

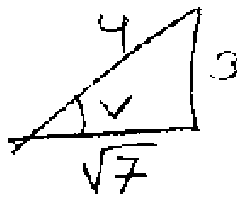
III: $x^2+2x=1-x \qquad x=-\frac{3}{2}\pm\sqrt{\frac{9}{4}+1} \qquad x=-\frac{3}{2}+\sqrt{\frac{13}{4}} \in III$

5a) $\ln 2^5 = \ln(\sqrt[3]{x}\sqrt{x}) = \ln x^{1/3+1/2} = \ln x^{5/6}$

$2^5 = x^{5/6}$ $\quad \underline{x = 2^6}$

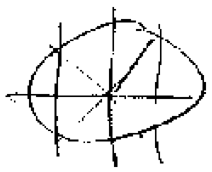
b) $2^x = t \quad 2t^2 = 8t + 10 \quad t = 2 \pm \sqrt{4+5} = 2 \pm 3$

$2^x > 0 \Rightarrow t = 5 \Rightarrow x\ln 2 = \ln 5 \quad \underline{x = \frac{\ln 5}{\ln 2}}$

6a) 

$\cos v = \sqrt{7}/4$

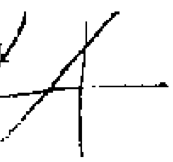
6b) $\sin^2 v = 1 - \cos^2 v \Rightarrow \cos^2 v = \frac{3}{4} \quad \cos v = \pm\frac{\sqrt{3}}{2}$



$v = \pm\frac{\pi}{3} + n2\pi, \; v = \pm\frac{2\pi}{3} + n2\pi$ (eller $v = \pm\frac{\pi}{3} + n\pi$)

6c) $\cos 2v = \cos(\frac{\pi}{2} - v - \frac{\pi}{3}) = \cos(\frac{\pi}{6} - v) \quad 2v = \pm(\frac{\pi}{6} - v) + n2\pi$

$v = \frac{\pi}{18} + n\frac{2\pi}{3}, \; v = -\frac{\pi}{6} + n2\pi$

7a) 

$y = \frac{4x}{3} + 4$

7b) $2\left((x - \frac{3}{2})^2 - \frac{9}{4}\right) + 5\left((y+1)^2 - 1\right) = 3$

$2(x - 3/2)^2 + 5(y+1)^2 = 9/2 + 5 + 3 = 25/2$

$\frac{(x-3/2)^2}{(5/2)^2} + \frac{(y+1)^2}{(\sqrt{5/2})^2} = 1$ centr $(3/2, -1)$ halvax $5/2, \sqrt{5/2}$

7c) $(x-a)^2 + (y - (3a+1))^2 = 8 \quad (1-a)^2 + (3a+1)^2 = 8$

$10a^2 + 4a - 6 = 0 \quad a = -\frac{1}{5} \pm \sqrt{\frac{1}{25} + \frac{3}{5}} = -\frac{1}{5} \pm \frac{4}{5} = -1, 3/5$

$(x+1)^2 + (y+2)^2 = 8, \; (x - 3/5)^2 + (y - 14/5)^2 = 8$

8a) $x(2\sqrt{x} - x + 4) = 0 \Leftrightarrow \underline{x = 0}, \; \sqrt{x} = 1 \pm \sqrt{1+4}$

$\Rightarrow \underline{x = (1+\sqrt{5})^2}$

8b) $f'(x) = 3\sqrt{x} - 2x + 4 \quad f'(4) = 2$

tgt: $y - 16 = 2(x-4)$ normal: $y - 16 = -\frac{1}{2}(x-4)$