Svar Matte intro 23/10 - 09

1a) $e^{1/2-1-\frac{2}{3}} = e^{-7/6}$

1b) $\frac{(y-x)(y+x)}{x^2y^2}\,\frac{xy}{y-x} = \frac{y+x}{xy}$

1c) $\frac{6ab+4b^2+a^2-2ab}{a^2-4b^2} = \frac{(a+2b)^2}{a^2-4b^2} = \frac{a+2b}{a-2b}$

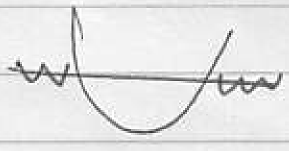
2a) $x^2=t \quad t=-\frac{3}{2}\pm\sqrt{\frac{9}{4}+4} = \frac{-3\pm 5}{2} = 1$ el. -4

$x=\pm 1$

2b) $3+2t = 1-2t+t^2 \quad t = 2\pm\sqrt{4+2}$

$1-t<0$ for $t=2+\sqrt{6}$

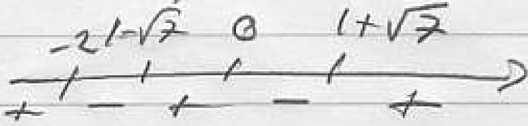
3a) $x=\frac{3}{2}\pm\sqrt{\frac{9}{4}-1} = \frac{3\pm\sqrt{5}}{2}$



$x\le \frac{3-\sqrt{5}}{2}$ el. $x\ge\frac{3+\sqrt{5}}{2}$

3b) $\frac{x^2+2x-3x-6-x}{x(x+2)} = \frac{x^2-2x-6}{x(x+2)} > 0$

$\frac{(x-(1-\sqrt{7}))(x-(1+\sqrt{7}))}{x(x+2)} > 0$



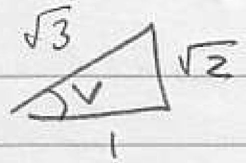
$x<-2$ eller $1-\sqrt{7}<x<0$ eller $x>1+\sqrt{7}$

4) $\pm 1, \pm 2, \pm 7, \pm 14 \quad (x+2)(\underbrace{x^2-2x-7}_{(x-(1+2\sqrt{2}))(x-(1-2\sqrt{2}))})$

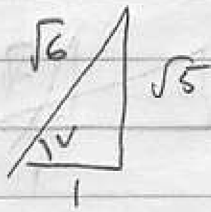
5a) $\ln\frac{x\cdot x^9}{x^4}=3 \iff \ln x^6 = 3 \iff x = (e^3)^{1/6} = \sqrt{e}$

5b) $e^a=t \quad t^2-\frac{4t}{3}-\frac{4}{3}=0 \quad t=\frac{2}{3}\pm\sqrt{\frac{4}{9}+\frac{4}{3}} = \frac{2\pm 4}{3}$

$a=\ln 2$

6a) 

$\cos v = \frac{1}{\sqrt{3}}$

6b) $\tan(2v) = \frac{2\tan v}{1-\tan^2 v}$ 

$= \pm \frac{2\sqrt{5}}{-4} = \pm \frac{\sqrt{5}}{2}$

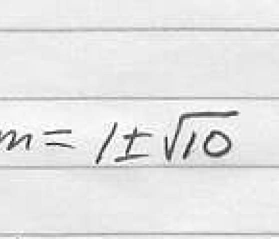
6c) $\sin v - \sin^2 v = 0 \qquad \sin v(1 - \sin v) = 0$

$v = n\pi$ eller $v = \frac{\pi}{2} + n2\pi$

7a) $y - 1 = -\frac{2}{3}(x+2) \qquad y = -\frac{2x}{3} - \frac{1}{3}$

7b) $(x+2)^2 + 2\left(y - \frac{3}{4}\right)^2 = \frac{7}{8} + 4 + \frac{9}{8} = 6$

$\frac{(x+2)^2}{(\sqrt{6})^2} + \frac{\left(y-\frac{3}{4}\right)^2}{(\sqrt{3})^2} = 1$

7c) $(x-1)^2 + (y+1)^2 = 2 \qquad y = -2x + m$ 

$x^2 - 2x + (-2x+m)^2 + 2(-2x+m) = 0$

$5x^2 + x(-2-4m-4) + m^2 + 2m = 0$

$x = \frac{3+2m}{5} \pm \sqrt{\left(\frac{3+2m}{5}\right)^2 - \frac{m^2+2m}{5}}$

$-m^2 + 2m + 9 = 0 \qquad m = 1 \pm \sqrt{10}$

8a) $f'(x) = \frac{2x\sqrt{2x} - (x^2+1)\frac{1}{\sqrt{2x}}}{2x} = \frac{3x^2-1}{2x\sqrt{2x}}$

$f'(2) = \frac{11}{8}$ tgt: $y - \frac{5}{2} = \frac{11}{8}(x-2)$

normal: $y - \frac{5}{2} = -\frac{8}{11}(x-2)$

8b) $x = \pm \frac{1}{\sqrt{3}}$