BI/KI LMA033/515
Svar matte B/C 20120112

③
$$\begin{bmatrix} -a & -4 & -6 \\ -1 & -a & -3 \\ 1 & 2 & 5-a \end{bmatrix} \begin{matrix} \hookleftarrow \\ \hookleftarrow \\ \textcircled{1}\textcircled{a} \end{matrix} \sim \begin{bmatrix} 1 & 2 & 5-a \\ 0 & 2a-4 & -6+5a-a^2 \\ 0 & 2-a & 2-a \end{bmatrix} \begin{matrix} \\ \\ \hookleftarrow 2 \end{matrix}$$

$$\begin{bmatrix} 1 & 2 & 5-a \\ 0 & 2-a & 2-a \\ 0 & 0 & -a^2+3a-2 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 5-a \\ 0 & 2-a & 2-a \\ 0 & 0 & -(a-1)(a-2) \end{bmatrix}$$

$a \neq 1, a \neq 2$: $x = y = z = 0$

$a = 1$: $\begin{bmatrix} 1 & 2 & 4 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad x = -2y - 4z = -2t$, $y = -t$, $z = t$

$a = 2$: $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad x = -2s - 3t$, $y = s$, $z = t$

④ Sätt in i $y = kx + m$:

$$\left.\begin{aligned} k\cdot(-1) + m &= 1 \\ k\cdot 1 + m &= 1 \\ k\cdot 2 + m &= 1 \\ k\cdot 3 + m &= 2 \end{aligned}\right\} \quad \underbrace{\begin{bmatrix} -1 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} k \\ m \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}}_{b}$$

$A^TAx = A^Tb$ (minsta kvadrat)

$$\begin{bmatrix} 15 & 5 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} k \\ m \end{bmatrix} = \begin{bmatrix} 8 \\ 5 \end{bmatrix} \begin{matrix} \hookleftarrow \\ \textcircled{-3} \end{matrix} \quad \begin{bmatrix} 0 & -7 \\ 5 & 4 \end{bmatrix} \begin{matrix} -7 \\ 5 \end{matrix}$$

$m = 1 \quad k = \frac{5-4m}{5} = \frac{1}{5} \quad y = \frac{1}{5}x + 1$

(5) $AX = B - 2X \Leftrightarrow AX + 2EX = B$

$\Leftrightarrow (A+2E)X \Leftrightarrow B \Leftrightarrow$ ~~$X = A^{-1}$~~

$\Leftrightarrow (A+2E)^{-1}(A+2E)X = (A+2E)^{-1}B$

$\Leftrightarrow \underbrace{EX}_{X} = (A+2E)^{-1}B \Leftrightarrow X = (A+2E)^{-1}B$

$(A+2E)^{-1}$:

$$\left[\begin{array}{cccccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 2 & -1 & 1 & 0 & 1 & 0 \\ 1 & 3 & -2 & 0 & 0 & 1 \end{array}\right] \begin{matrix} \\ (-2) \\ (-1) \end{matrix}$$

$$\left[\begin{array}{cccccc} 1 & -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 4 & -3 & -1 & 0 & 1 \end{array}\right] \begin{matrix} \\ (1) \\ (-4) \end{matrix} \sim \left[\begin{array}{cccccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & 1 & 7 & -4 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 5 & -3 & 1 \\ 0 & 0 & 1 & 7 & -4 & 1 \end{array}\right] \quad X = \begin{bmatrix} -1 & 1 & 0 \\ 5 & -3 & 1 \\ 7 & -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 0 \\ 6 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 6 \\ -1 & 8 \end{bmatrix}$$

(6) (a)

$$\left[\begin{array}{cccc} 1 & 2 & -2 & 1 \\ 1 & -1 & 4 & 19 \end{array}\right] \begin{matrix} (-1) \\ \downarrow \end{matrix} \sim \left[\begin{array}{cccc} 1 & 2 & -2 & 1 \\ 0 & -3 & 6 & 18 \end{array}\right] (-1/3)$$

$$\sim \left[\begin{array}{cccc} 1 & 2 & -2 & 1 \\ 0 & 1 & -2 & -6 \end{array}\right] \begin{matrix} (9) \\ (-2) \end{matrix} \sim \left[\begin{array}{cccc} 1 & 0 & 2 & 13 \\ 0 & 1 & -2 & -6 \end{array}\right]$$

$$\begin{cases} x = 13 - 2t \\ y = -6 + 2t \\ z = t \end{cases}$$

richtvektor $\begin{bmatrix} -2 \\ 2 \\ 1 \end{bmatrix}$

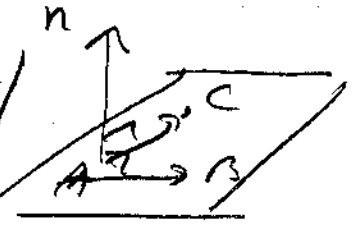
(6b) $w \cdot u = 1 \quad w_p \cdot u = x\|u\|^2 + y\, v \cdot u = 2x + 4y$

$w \cdot v = 5 \quad w_p \cdot v = x\, u \cdot v + y\|v\|^2 = 4x + 11y$

$$\begin{cases} 2x + 4y = 1 & |-2| \\ 4x + 11y = 5 & \downarrow \end{cases}$$

$3y = 3 \quad y = 1 \quad x = -3/2 \quad w_p = (\frac{3}{2}, 1, \frac{1}{2})$

(7a) $n = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} e_x & e_y & e_z \\ 1 & 0 & -2 \\ 0 & -2 & 2 \end{vmatrix}$



$= (-4, -2, -2)$

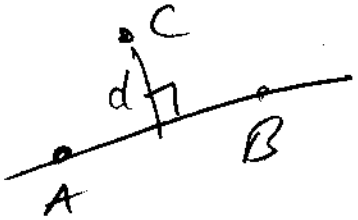
$-4(x-1) - 2(y-1) - 2(z-2) = 0$

$2x + y + z - 5 = 0$

(7b) $V = \left| \frac{1}{6} (\overrightarrow{AB} \times \overrightarrow{AC}) \cdot \overrightarrow{AD} \right|$

$= \frac{1}{6} \left| (-4, -2, -2) \cdot (4, -2, -2) \right| = \frac{1}{6} \cdot |-8| = \frac{4}{3}$

(7c)



$d = \frac{|\overrightarrow{AB} \times \overrightarrow{AC}|}{|\overrightarrow{AB}|} = \frac{|(-4, -2, -2)|}{|(1, 0, -2)|} = \frac{\sqrt{24}}{\sqrt{5}}$

(8a) $x = 1 + t$
$y = -3t$
$z = 1$

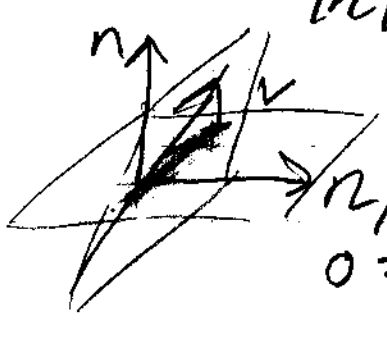
$7 + 2(1+t) - (-3t) + 1 = 0 \Leftrightarrow 10 + 5t = 0 \Leftrightarrow t = -2$

plug $(-1, 6, 1)$

(8b) $d = \frac{|7 + 2x - y + z|}{\sqrt{2^2 + 1^2 + 1^2}} = \frac{7+2+1}{\sqrt{6}} = \frac{10}{\sqrt{6}}$

(alt $d = \frac{|\overrightarrow{P_0P_1} \cdot n|}{|n|} = \frac{|((1,0,1) - (0,0,[redacted]) \cdot (2,-1,1)|}{|(2,-1,1)|}$)

(8c)



$n_1 = n \times v = \begin{vmatrix} e_x & e_y & e_z \\ 2 & -1 & 1 \\ 1 & -3 & 0 \end{vmatrix} = (3, 1, -5)$

$0 = 3(x-1) + [redacted] - 5(z-1) = 3x + y - 5z + 2$