

③
$$\begin{bmatrix} 1-a & 3 & 3 \\ -3 & -5-a & -3 \\ 3 & 3 & 1-a \end{bmatrix} \begin{matrix} \leftarrow \\ \leftarrow \\ \textcircled{1} \frac{a-1}{3} \end{matrix} \sim \begin{bmatrix} 0 & a+2 & \frac{(10-a)(8+a)}{3} \\ 0 & -2-a & -2-a \\ 3 & 3 & 1-a \end{bmatrix} \begin{matrix} \leftarrow \\ \leftarrow \\ \textcircled{1} \end{matrix}$$

$$\begin{bmatrix} 0 & 0 & \frac{2-a^2-a}{3} \\ 0 & -2-a & -2-a \\ 3 & 3 & 1-a \end{bmatrix} \quad a \neq 1, a \neq -2 : x=y=z=0$$

$a=1 : \begin{cases} x = t \\ y = -t \\ z = t \end{cases} \quad t \text{ godt.}$

$a=-2 : \begin{cases} x = -s - t \\ y = t \\ z = s \end{cases} \quad s, t \text{ godt.}$

④ $y = kx + m$

$1 = m$
 $1 = k + m$
 $2 = 2k + m$
 $2 = 3k + m$

$$\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 2 & 1 \\ 3 & 1 \end{bmatrix} \begin{matrix} \\ \\ \\ A \end{matrix} \begin{matrix} \\ \\ \\ X \end{matrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{matrix} \begin{matrix} \\ \\ \\ b \end{matrix} \quad A^T A x = A^T b$$

$$\begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \end{bmatrix} X = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 14 & 6 \\ 6 & 4 \end{bmatrix} X = \begin{bmatrix} 11 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 14 & 6 & 11 \\ 3 & 2 & 3 \end{bmatrix} \begin{matrix} \leftarrow \\ \textcircled{-3} \end{matrix} \begin{bmatrix} 5 & 0 & 2 \\ 3 & 2 & 3 \end{bmatrix} \quad k = \frac{2}{5} \quad m = \frac{3 - \frac{6}{5}}{2} = \frac{9}{10}$$

⑤ $X(E-A) = B \Leftrightarrow X = B(E-A)^{-1}$

$$\begin{bmatrix} D & 0 & 2 & 1 & 0 & 0 \\ 3 & 0 & -4 & 0 & 1 & 0 \\ -2 & 3 & -3 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} \leftarrow \\ \leftarrow \\ \textcircled{3/2} \end{matrix} \sim \begin{bmatrix} 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 9/2 & -7/2 & 0 & 1 & 3/2 \\ -2 & 3 & -3 & 0 & 0 & 1 \end{bmatrix} \begin{matrix} \leftarrow \\ \leftarrow \\ \textcircled{-2/3} \end{matrix} \sim \begin{bmatrix} 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 9/2 & -7/2 & 0 & 1 & 3/2 \\ -2 & 0 & 8/3 & 0 & 2/3 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} 0 & 0 & 2 & 1 & 0 & 0 \\ 0 & 9/2 & 0 & 17/4 & 1 & 3/2 \\ -2 & 0 & 0 & -4/3 & -2/3 & 0 \end{bmatrix} \quad (E-A)^{-1} = \begin{bmatrix} 2/3 & 1/3 & 0 \\ 17/18 & 2/9 & 1/3 \\ 1/2 & 0 & 0 \end{bmatrix} \quad X = \begin{bmatrix} 23/9 & 7/9 & 2/3 \\ 7/6 & 1/3 & 0 \end{bmatrix}$$

$$6a \quad |u+v|^2 = 7, |u-2v|^2 = 7 \quad u \cdot v = 1$$

$$|u|^2 + 2u \cdot v + |v|^2 = 7, |u|^2 - 4u \cdot v + 4|v|^2 = 7$$

$$\left. \begin{array}{l} |u|^2 + |v|^2 = 5 \\ |u|^2 + 4|v|^2 = 11 \end{array} \right\} \textcircled{-} \quad \begin{array}{l} 3|v|^2 = 6 \quad |v| = \sqrt{2} \\ |u|^2 = 3 \quad |u| = \sqrt{3} \end{array}$$

$$6b \quad u = tb, v = sc \quad (a - tb - sc) \cdot b = 0 = (a - tb - sc) \cdot c$$

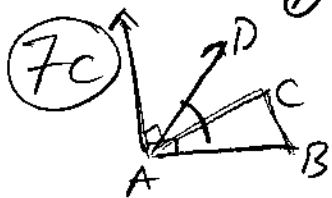
$$\begin{cases} (2 - 3t - s) \cdot 3 + (-t) \cdot 1 + (1 + t + s) \cdot (-1) = 0 \\ (2 - 3t - s) \cdot 1 + (-t) \cdot 0 + (1 + t + s) \cdot (-1) = 0 \end{cases} \Leftrightarrow \begin{cases} 5 - 11t - 4s = 0 \\ -4t - 2s = 0 \end{cases}$$

$$t = 1, s = -3/2$$

$$7a \quad \vec{AB} \times \vec{AC} = \begin{vmatrix} e_x & e_y & e_z \\ 1 & 0 & -2 \\ -3 & -1 & 0 \end{vmatrix} = (-2, 6, -1)$$

$$(x-1) \cdot (-2) + (y-1) \cdot 6 + (z-2) \cdot (-1) = 0 \quad -2x + 6y - z - 2 = 0$$

$$7b \quad V = \frac{1}{6} |\vec{AD} \cdot (\vec{AB} \times \vec{AC})| = \frac{1}{6} |(-1, -2, 1) \cdot (-2, 6, -1)| = \frac{11}{6}$$



$$7c \quad \cos \theta = \frac{\vec{AD} \cdot (\vec{AB} \times \vec{AC})}{|\vec{AD}| |\vec{AB} \times \vec{AC}|} = \frac{-11}{\sqrt{6} \sqrt{41}}$$

$$\text{winkel} = \frac{\pi}{2} - \arccos \frac{11}{\sqrt{6} \cdot \sqrt{41}}$$

$$8a \quad L: \begin{array}{l} x = 1 - 2t \\ y = 3 - 5t \\ z = 1 - t \end{array} \quad \begin{array}{l} 3(1 - 2t) - (3 - 5t) + 4 = 0 \\ t = 4 \quad (-7; -17; -3) \end{array}$$

$$8b \quad |\vec{P_0 P} \cdot n| = 2 |n| \quad \text{t.ex } P_0 = (0; 0; -2)$$

$$|(1 - 2t) \cdot (-2) + (3 - 5t) \cdot 6 + (3 - t) \cdot (-1)| = 2 \sqrt{41}$$

$$|15 - 25t| = 2 \sqrt{41} \Leftrightarrow t = \frac{15 \pm 2 \sqrt{41}}{25}$$

$$8c \quad x = 1 - 2t, y = 3 + 6t, z = 1 - t \quad 3(1 - 2t) - (3 + 6t) + 4 = 0 \quad t = 1/3$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -7 \\ -17 \\ -3 \end{bmatrix} + t \begin{bmatrix} 13 \\ 5 \\ 2/3 \end{bmatrix} - \begin{bmatrix} -7 \\ -17 \\ -3 \end{bmatrix}$$