Uppgift 1(a). För olika t_i-värden skall vi lösa $f(s) = s + \ln(s) + t_i - 1 = 0$ för att få värdet på $s(t_i)$. Då $t = 0$ gäller att $s = 1$.

```
f=@(s,t)s+log(s)+t-1;
t=linspace(0,10); s=zeros(size(t));
s(1)=1;
for i=2:length(t)
    s(i)=fzero(@(s)f(s,t(i)),s(i-1));
end
plot(t,s,'r')
```

(b). Vi har $s'(t) + \frac{s'(t)}{s(t)} = -1$, dvs. $s'(t) = -\frac{s(t)}{1+s(t)}$ samt $s(0) = 1$.

```
f=@(t,s)-s/(1+s); s0=1;
[t,s]=ode45(f,t,s0);
plot(t,s,'b')
```

Uppgift 2. Se Lay kapitel 5.

Uppgift 3(a). Egenvärdesproblemet:

$$\det(\boldsymbol{A} - \lambda \boldsymbol{I}) = \left| \begin{bmatrix} -4-\lambda & 1 \\ 3 & -2-\lambda \end{bmatrix} \right| = (-4-\lambda)(-2-\lambda) - 3 =$$

$$= \lambda^2 + 6\lambda + 5 = (\lambda+3)^2 - 4 = 0 \quad \Rightarrow \quad \lambda_1 = -5,\ \lambda_2 = -1$$

Egenvektorn som hör ihop med $\lambda_1 = -5$:

$$(\boldsymbol{A} - \lambda_1 \boldsymbol{I})\boldsymbol{v} = (\boldsymbol{A} + 5\boldsymbol{I})\boldsymbol{v} = \begin{bmatrix} 1 & 1 \\ 3 & 3 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ ger } \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Egenvektorn som hör ihop med $\lambda_2 = -1$:

$$(\boldsymbol{A} - \lambda_2 \boldsymbol{I})\boldsymbol{v} = (\boldsymbol{A} + \boldsymbol{I})\boldsymbol{v} = \begin{bmatrix} -3 & 1 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ ger } \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Lösningen till ODE-systemet:

$$\boldsymbol{u}(t) = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} \exp(-5t) + c_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} \exp(-t)$$

$$\boldsymbol{u}(0) = c_1 \begin{bmatrix} 1 \\ -1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \Rightarrow \boldsymbol{c} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$\boldsymbol{u}(t) = 2 \begin{bmatrix} 1 \\ -1 \end{bmatrix} \exp(-5t) + \begin{bmatrix} 1 \\ 3 \end{bmatrix} \exp(-t)$$

(b). Se laboration 4 eller Lay kapitel 5.

(c). Euler framåt: $\boldsymbol{u}_{n+1} = \boldsymbol{u}_n + h\boldsymbol{A}\boldsymbol{u}_n = (\boldsymbol{I} + h\boldsymbol{A})\boldsymbol{u}_n, n = 0, 1, \cdots, \boldsymbol{u}_0 = [3 \;\; 1]^T$

$$\boldsymbol{u}_1 = (\boldsymbol{I} + 0.1\boldsymbol{A})\boldsymbol{u}_0 = \begin{bmatrix} 0.6 & 0.1 \\ 0.3 & 0.8 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.9 \\ 1.7 \end{bmatrix}$$

Uppgift 4(a).

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{bmatrix} x_1 + x_2 \exp(x_1 x_2) + 1 \\ x_1^3 + x_2 - 1 \end{bmatrix}, \quad \boldsymbol{Df}(\boldsymbol{x}) = \begin{bmatrix} 1 + x_2^2 \exp(x_1 x_2) & (1 + x_1 x_2)\exp(x_1 x_2) \\ 3x_1^2 & 1 \end{bmatrix}$$

(b). Se laboration 5.

(c). Newtons metod: $\boldsymbol{x}_{k+1} = \boldsymbol{x}_k + \boldsymbol{d}_k$, där $\boldsymbol{Df}(\boldsymbol{x}_k)\,\boldsymbol{d}_k = -\boldsymbol{f}(\boldsymbol{x}_k)$

$$\boldsymbol{x}_0 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}: \quad \boldsymbol{Df}(\boldsymbol{x}_0)\,\boldsymbol{d}_0 = -\boldsymbol{f}(\boldsymbol{x}_0) \Leftrightarrow \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} d_{01} \\ d_{02} \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \end{bmatrix} \Leftrightarrow \boldsymbol{d}_0 = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$\boldsymbol{x}_1 = \boldsymbol{x}_0 + \boldsymbol{d}_0 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

(d).

```
f1=@(x1,x2)x1+x2.*exp(x1.*x2)+1;
f2=@(x1,x2)x1.^3+x2-1;
f=@(x)[f1(x(1),x(2));f2(x(1),x(2))];
Df=@(x)[1+x(2)^2*exp(x(1)*x(2)) (1+x(1)*x(2))*exp(x(1)*x(2));3*x(1)^2 1];
x=[0;1];
kmax=10; tol=0.5e-8;
for k=1:kmax
    d=-Df(x)\f(x);
    x=x+d;
    disp([x' norm(d)])
    if norm(d)<tol, break, end
end
```

Uppgift 5(a). Vi har $f(\boldsymbol{x}) = -x_1^3 + 5x_1x_2 - 3x_2^3 - 1$ och

$$\boldsymbol{\nabla} f(\boldsymbol{x}) = \begin{bmatrix} -3x_1^2 + 5x_2 \\ 5x_1 - 9x_2^2 \end{bmatrix} \qquad \boldsymbol{H}(\boldsymbol{x}) = \begin{bmatrix} -6x_1 & 5 \\ 5 & -18x_2 \end{bmatrix}$$

(b).

```
f=@(x1,x2)-x1.^3+5*x1.*x2-3*x2.^3-1;
dfdx1=@(x1,x2)-3*x1.^2+5*x2;
dfdx2=@(x1,x2)5*x1-9*x2.^2;
gradf=@(x)[dfdx1(x(1),x(2))
           dfdx2(x(1),x(2))];
H=@(x)[-6*x(1)  5
        5      -18*x(2)];
x1min=-4; x1max=4; x2min=-4; x2max=4;
x1=linspace(x1min,x1max,50); x2=linspace(x2min,x2max,50);
[X1,X2]=meshgrid(x1,x2); Z=f(X1,X2);
figure(1), clf
surf(x1,x2,Z), xlabel('x'), ylabel('y')
figure(2), clf
```

```
contour(x1,x2,Z,linspace(-40,40,50)), xlabel('x'), ylabel('y')
hold on
contour(x1,x2,dfdx1(X1,X2),[0 0],'k')
contour(x1,x2,dfdx2(X1,X2),[0 0],'m')
axis equal, grid on
```

(c).

```
x0=ginput(1)';
x=fsolve(gradf,x0)
plot(x(1),x(2),'ro')
eg=eig(H(x))
```

Uppgift 6(a). Inför nät med steglängden h i rumsled och ersätt u''_{xx} med D_+D_-. Låt $u_i(t)$ beteckna approximationen av $u(x_i,t)$. För differentialekvationen får vi

$$u_i'(t) = \frac{\kappa}{h^2}(u_{i+1}(t) - 2u_i(t) + u_{i-1}(t)) + f(x_i,t) + C(u_{omg}^4 - u_i(t)^4), \quad i = 1, \cdots, n, \quad 0 < t < T$$

och randvillkoren ger

$$u_0(t) = g_0, \quad u_{n+1}(t) = g_L, \quad 0 \le t \le T$$

Begynnelsevillkoren blir

$$u_i(0) = u_0(x_i), \quad i = 1, \cdots, n$$

Begynnelsevärdesproblemet för ODE blir

$$\begin{cases} \boldsymbol{U}'(t) = \frac{\kappa}{h^2}(\boldsymbol{b}(t) - \boldsymbol{A}\boldsymbol{U}(t)) + \boldsymbol{F}(t) + C(u_{omg}^4 - \boldsymbol{U}^4(t)), \quad 0 < t < T \\ \boldsymbol{U}(0) = \boldsymbol{U}_0 \end{cases}$$

där

$$\boldsymbol{b} = \begin{bmatrix} g_0 \\ 0 \\ \vdots \\ 0 \\ g_L \end{bmatrix} \quad \boldsymbol{A} = \begin{bmatrix} 2 & -1 & & & \\ -1 & 2 & -1 & & \\ & \ddots & \ddots & \ddots & \\ & & -1 & 2 & -1 \\ & & & -1 & 2 \end{bmatrix} \quad \boldsymbol{F}(t) = \begin{bmatrix} f(x_1,t) \\ f(x_2,t) \\ \vdots \\ f(x_{n-1},t) \\ f(x_n,t) \end{bmatrix}$$

och

$$\boldsymbol{U}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_{n-1}(t) \\ u_n(t) \end{bmatrix} \quad \boldsymbol{U}^4(t) = \begin{bmatrix} u_1(t)^4 \\ u_2(t)^4 \\ \vdots \\ u_{n-1}(t)^4 \\ u_n(t)^4 \end{bmatrix} \quad \boldsymbol{U}_0 = \begin{bmatrix} u_0(x_1) \\ u_0(x_2) \\ \vdots \\ u_0(x_{n-1}) \\ u_0(x_n) \end{bmatrix}$$

(b).

```
kappa=...; C=...; uomg=...; L=...; g0=...; gL=...; T=...;
f=@(x,t)...; u0=@(x)...;
n=30;
h=L/(n+1); xi=h*[1:n]'; tspan=linspace(0,T,n+1); U0=u0(xi);
A=spdiags(ones(n,1)*[-1 2 -1],[-1 0 1],n,n); b=[g0;zeros(n-2,1);gL];
traad=@(t,u)kappa/h^2*(b-A*u)+f(xi,t)+C*(uomg^4-u.^4);
[t,U]=ode45(traad,tspan,U0);
x=[0;xi;L]; U=[g0*ones(size(t)),U,gL*ones(size(t))];  % Kantar med randvärden
surf(x,t,U)
xlabel('x'), ylabel('t')
```