Lösningar LMA 400

1. a) $D\left(\left((f(x))^2-1\right)^2\right) = 2\left((f(x))^2-1\right)\cdot 2f(x)\cdot f'(x)$

$$\Rightarrow \frac{d}{dx}\left((f(x))^2-1\right)^2\Big|_{x=2} = 2(2^2-1)\cdot 2\cdot 2\cdot 3 = 72$$

b) $$\int \frac{x+\ldots}{x^2+5x+6}\,dx = \int\left(\frac{A}{x+2}+\frac{B}{x+3}\right)dx = \int\left(\frac{-1}{x+2}+\frac{2}{x+3}\right)dx =$$

$$= 2\ln|x+3| - \ln|x+2| + C$$

c) $$\int \frac{\cos x}{1-\cos^2 x}\,dx = \int \frac{\cos x}{\sin^2 x}\,dx = \begin{bmatrix} \sin x = t \\ dt = \cos x\,dx \end{bmatrix} =$$

$$= \int \frac{1}{t^2}\,dt = -\frac{1}{t} + C = -\frac{1}{\sin x} + C$$

d) KE: $r^2 - 6r + 9 = 0 \Leftrightarrow r_1 = r_2 = 3$

$\Rightarrow y(x) = (C_1 x + C_2)e^{3x}$; $y(0) = 2$ ger $C_2 = 2$

$\Rightarrow y(x) = (C_1 x + 2)e^{3x}$; $y'(0) = -3$ ger $C_1 = -9$

$\Rightarrow y'(x) = C_1 e^{3x} + 3(C_1 x + 2)e^{3x}$

$\Rightarrow y(x) = (-9x+2)e^{3x}$

e) $$\lim_{x\to-\infty} x\left(\arctan x + \frac{\pi}{2}\right) = \lim_{x\to-\infty} \frac{\arctan x + \pi/2}{1/x} \overset{H}{=}$$

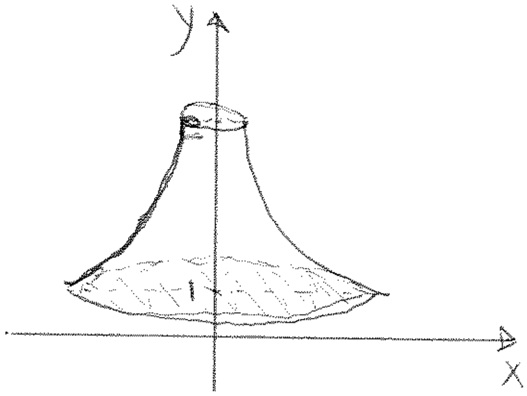
$$= \lim_{x\to-\infty} \frac{\frac{1}{1+x^2}}{-\frac{1}{x^2}} = \lim_{x\to-\infty} -\frac{x^2}{1+x^2} = -1.$$

2. a) Se boken.

b) $y = \frac{1}{x^2} \Rightarrow x^2 = \frac{1}{y}$.

$$V = \pi \int_1^e x^2 \, dy = \pi \int_1^e \frac{1}{y} \, dy$$

$$= \pi \Big[\ln y \Big]_1^e = \pi \text{ v.e.}$$



---

3. a) Se boken.

b) $f(x) = \arcsin x + \arccos x$

$$\Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{1-x^2}} = 0$$

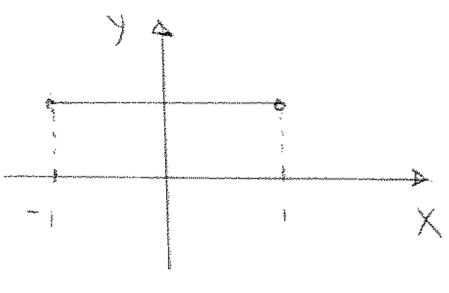
$\Rightarrow$ f är en konstant funktion

$$f(-1) = \arcsin(-1) + \arccos(-1) = -\frac{\pi}{2} + \pi = \frac{\pi}{2}$$

$$f(1) = \arcsin 1 + \arccos 1 = \frac{\pi}{2} + 0 = \frac{\pi}{2}$$

så

$f(x) = \frac{\pi}{2}$ då $x \in [-1, 1]$



4. a) $x \neq \pm 1$

$\lim_{x \to 1^+} \frac{x^3}{x^4-1} = "+\infty"$, $\lim_{x \to 1^-} \frac{x^3}{x^4-1} = "-\infty"$ } $x = 1$ lodrät asymptot

$\lim_{x \to -1^+} \frac{x^3}{x^4-1} = "+\infty"$, $\lim_{x \to -1^-} \frac{x^3}{x^4-1} = "-\infty"$ } $x = -1$ lodrät asymptot

$\lim_{x \to \infty} f(x) = 0$ och $\lim_{x \to -\infty} f(x) = 0$ $\Rightarrow y = 0$ vågrät asymptot då $x \to \pm\infty$

b) $$f'(x) = \frac{3x^2(x^4-1) - x^3 \cdot 4x^3}{(x^4-1)^2} = \frac{3x^6 - 3x^2 - 4x^6}{(x^4-1)^2} =$$

$$= \frac{-x^2(x^4+3)}{(x^4-1)^2}$$

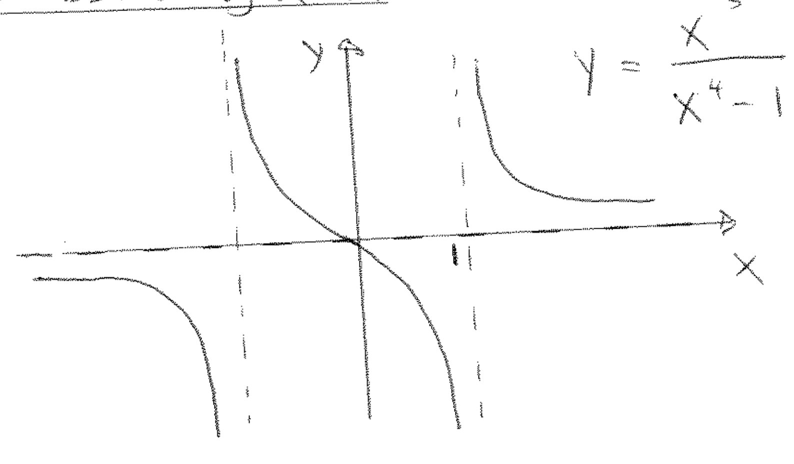
$f'(x) = 0$ då $x = 0$

$f'(x) < 0$ då $x < 0$ och $f'(x) < 0$ då $x > 0$

och vi har en terasspunkt i origo

c) Vidare ser vi att $f'(x) < 0$ för alla x där f är definerad

Skissera grafen.



5. a) $I' + \mu I = 0$

I.F. $= e^{\mu x}$

$$\Rightarrow I' e^{\mu x} + e^{\mu x} \cdot \mu I = 0 \Leftrightarrow (e^{\mu x} I)' = 0$$

$$\Leftrightarrow e^{\mu x} I = C \Leftrightarrow \underline{\underline{I(x) = Ce^{-\mu x}}}$$

$I(0) = I_0$ ger $C = I_0$, så $\underline{I(x) = I_0 e^{-\mu x}}$

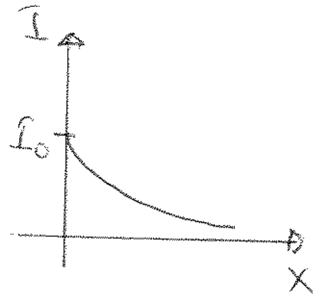
---

b) Av texten får vi att $I(1) = 0{,}5 I_0$

$$\Rightarrow \left.\begin{array}{l} I(1) = I_0 e^{-\mu \cdot 1} \\ I(1) = 0{,}5 I_0 \end{array}\right\} \Leftrightarrow \frac{1}{2}\cancel{I_0} = \cancel{I_0} e^{-\mu}$$

$$\Rightarrow -\mu = \ln \frac{1}{2} = \underbrace{\ln 1}_{=0} - \ln 2 \Rightarrow \mu = \ln 2$$

Så, $I(x) = I_0 (e^{-\ln 2})^x = I_0 \left(\frac{1}{2}\right)^x$



---

c) Absorberar 87,5% innebär att 12,5% går igenom väggen.

$$\Rightarrow 0{,}125 \cancel{I_0} = \cancel{I_0} \left(\frac{1}{2}\right)^x$$

$$\Rightarrow \frac{1}{8} = \left(\frac{1}{2}\right)^x \Rightarrow \left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^x \Rightarrow$$

$$\underline{\underline{x = 3 \text{ cm.}}}$$

6. a) Sant.

$y'(0) = 0 \Rightarrow y$ kontinuerlig i 0.

$$\left.\begin{array}{l} y' < 0 \text{ då } x < 0 \\ y' > 0 \text{ då } x > 0 \end{array}\right\} \Rightarrow \text{lok. min. i } x = 0.$$

---

b) Sant.

$\sum_{i=1}^{n} \frac{i}{n^2} = \sum_{i=1}^{n} \frac{i}{n} \cdot \frac{1}{n}$ kan betraktas som en

Riemannsumma för $\int_0^1 x\,dx$ och

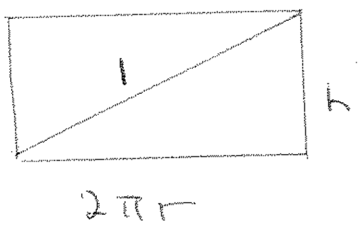
$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i}{n} \cdot \frac{1}{n} = \int_0^1 x\,dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}.$$

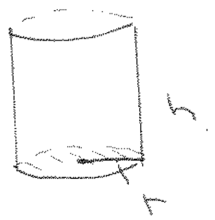
Alt. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{i}{n^2} = \lim_{n\to\infty} \frac{1}{n^2} \sum_{i=1}^{n} i =$

$$= \lim_{n\to\infty} \frac{1}{n^2} \cdot \frac{n(n+1)}{2} = \frac{1}{2} \lim_{n\to\infty} \frac{n^2+n}{n^2} =$$

$$= \frac{1}{2} \lim_{n\to\infty} \frac{\cancel{n^2}(1+\overbrace{1/n}^{\to 0})}{\cancel{n^2}} = \frac{1}{2} \cdot 1 = \frac{1}{2}.$$

7.





Omkretsen av cylinder-plattan är lika med längden, $2\pi r$, av rektangeln

Pyth. sats : $(2\pi r)^2 + h^2 = 1^2 \Leftrightarrow r^2 = \dfrac{1-h^2}{4\pi^2}$

<u>Volymen blir då:</u>

$$V(h) = \pi\left(\frac{1-h^2}{4\pi^2}\right)h = \frac{h-h^3}{4\pi}$$

$$\Rightarrow V'(h) = \frac{1}{4\pi}(1-3h^2) = 0 \text{ då } h = \frac{1}{\sqrt{3}}$$

$$V(0) = 0 \ ; \ \underbrace{V\left(\frac{1}{\sqrt{3}}\right) = \frac{1}{6\sqrt{3}\pi}}_{\text{Största värdet.}} \ ; \ V(1) = 0$$

---

8. $$\int_0^\infty \left(\frac{2x}{x^2+1} - \frac{c}{x+1}\right) dx = \lim_{t\to\infty}\Big[\ln(x^2+1) - c\ln(x+1)\Big]_0^t$$

$$= \lim_{t\to\infty}\left[\ln\frac{x^2+1}{(x+1)^c}\right]_0^t = \lim_{t\to\infty} \ln\frac{t^2+1}{(t+1)^c} =$$

$$= \ln\left(\lim_{t\to\infty}\frac{t^2+1}{(t+1)^c}\right)$$

- Om $c < 2$ så divergerar I mot ∞
- Om $c = 2$ så konvergerar I mot $\ln 1 = 0$
- Om $c > 2$ så divergerar I mot $-\infty$