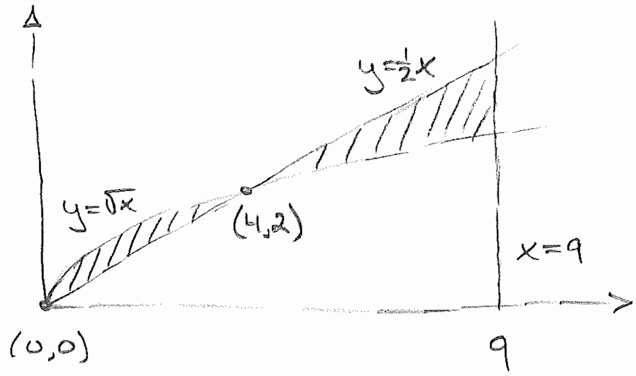
5.5.59) Beräkna $\int_1^2 \frac{e^{1/x}}{x^2}\,dx$

Lösn: $\int_1^2 \frac{e^{1/x}}{x^2}\,dx = \left\{ \begin{array}{l} u=\frac{1}{x},\ du=-\frac{1}{x^2}dx \\ x=1 \Rightarrow u=1,\ x=2 \Rightarrow u=\frac{1}{2} \end{array} \right\}$

$= -\int_1^{1/2} e^u\,du = \int_{1/2}^{1} e^u\,du = \left[e^u\right]_{1/2}^{1} = e - \sqrt{e}$

6.1.27) Skissa området mellan $y=\sqrt{x}$, $y=\frac{1}{2}x$ och $x=9$ och bestäm dess area.

Lösn:



Skärningspunkt: $\sqrt{x} = \frac{1}{2}x \Rightarrow x^2 - 4x = 0 \Rightarrow \begin{cases} x=0 \\ x=4 \end{cases}$

$$A = \int_0^9 \left|\sqrt{x} - \tfrac{1}{2}x\right| dx = \int_0^4 \left(\sqrt{x} - \tfrac{1}{2}x\right) dx + \int_4^9 \left(\tfrac{1}{2}x - \sqrt{x}\right) dx$$

$$= \left[\tfrac{2}{3}x^{3/2} - \tfrac{1}{4}x^2\right]_0^4 + \left[\tfrac{1}{4}x^2 - \tfrac{2}{3}x^{3/2}\right]_4^9$$

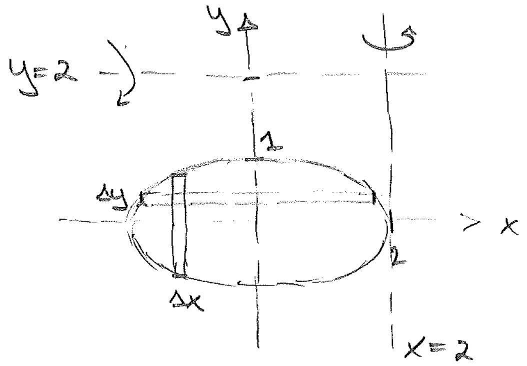
$$= \frac{16}{3} - 4 + \frac{81}{4} - 18 - \left(4 - \frac{16}{3}\right)$$

$$= \frac{81}{4} + \frac{32}{3} - 26 = \frac{59}{12}$$

6.2.33) Teckna integralen för volymen då kurvan $x^2+4y^2=4$ roteras kring

a) $y=2$ b) $x=2$

Lösn: $x^2+4y^2=4$ beskriver ellips



$$\frac{x^2}{2^2}+y^2=1$$

a) $y=\pm\sqrt{1-x^2/4}$

$$\Delta V=\left[\pi\left(2-\left(-\sqrt{1-x^2/4}\right)\right)^2-\pi\left(2-\sqrt{1-x^2/4}\right)^2\right]\Delta x$$

$$\Rightarrow V=\int_{-2}^{2}\pi\left[\left(2+\sqrt{1-x^2/4}\right)^2-\left(2-\sqrt{1-x^2/4}\right)^2\right]dx$$

$$=\int_{-2}^{2}8\pi\sqrt{1-x^2/4}\,dx=16\pi\int_0^2\sqrt{1-x^2/4}\,dx$$

b) $x=\pm\sqrt{4-4y^2}$

$$\Delta V=\left[\pi\left(2-\left(-\sqrt{4-4y^2}\right)\right)^2-\pi\left(2-\sqrt{4-4y^2}\right)^2\right]\Delta y$$

$$\Rightarrow V=\int_{-1}^{1}\pi\left[\left(2+\sqrt{4-4y^2}\right)^2-\left(2-\sqrt{4-4y^2}\right)^2\right]dy$$

$$=\int_{-1}^{1}8\pi\sqrt{4-4y^2}\,dy=16\pi\int_0^1\sqrt{4-4y^2}\,dy$$

NB: Integralerna i a) och b) är identiska

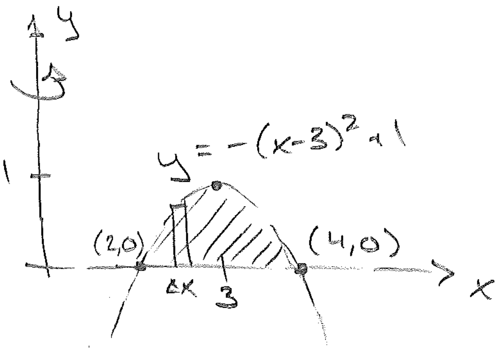
$$\int_0^2 \sqrt{1-x^2/4}\,dx = \int_0^2 \tfrac{1}{2}\sqrt{4-x^2}\,dx = \left\{ y = \tfrac{1}{2}x,\ dy = \tfrac{1}{2}dx \right\}$$

$$= \int_0^1 \sqrt{4-4y^2}\,dy$$

6.3.37) Bestäm volymen då regionen mellan $y = -x^2+6x-8$ och $y=0$ roteras kring y-axeln.

Lösn.:

$$y = -x^2+6x-8 = -(x-3)^2+1$$



$$y=0 \Rightarrow \begin{cases} x=2 \\ x=4 \end{cases}$$

Använd cylindriska skal!

$$V = \int_2^4 2\pi x\cdot(-x^2+6x-8)\,dx$$

$$= 2\pi\int_2^4 (-x^3+6x^2-8x)\,dx = 2\pi\left[-\tfrac{1}{4}x^4+2x^3-4x^2\right]_2^4$$

$$= 2\pi\left[-64+128-64-(-4+16-16)\right] = 8\pi$$

7.1.23) Beräkna $\int_0^{1/2} x \cos \pi x \, dx$

Lösn: $\int_0^{1/2} x \cos \pi x \overset{PI}{=} \left[\frac{1}{\pi} x \sin \pi x\right]_0^{1/2} - \frac{1}{\pi}\int_0^{1/2} \sin \pi x \, dx$

$$= \frac{1}{2\pi} \underbrace{\sin\frac{\pi}{2}}_{=1} - \frac{1}{\pi}\left[-\frac{1}{\pi}\cos \pi x\right]_0^{1/2}$$

$$= \frac{1}{2\pi} + \frac{1}{\pi^2}\left[\cos\frac{\pi}{2} - \cos 0\right] = \frac{1}{2\pi} - \frac{1}{\pi^2}$$

7.2.29) Bestäm $\int \tan^3 x \sec^6 x \, dx$

Lösn: $\sec x = \frac{1}{\cos x}$ och $1 + \tan^2 x = \frac{1}{\cos^2 x}$

$$\int \tan^3 x \frac{1}{\cos^6 x} dx = \int \tan^3 x (1+\tan^2 x)^2 \frac{1}{\cos^2 x} dx$$

$$= \left\{ \begin{array}{l} u = \tan x \\ du = \frac{1}{\cos^2 x} dx \end{array} \right\} = \int u^3 (1+u^2)^2 \, du$$

$$= \int (u^3 + 2u^5 + u^7) \, du = \frac{1}{4}u^4 + \frac{1}{3}u^6 + \frac{1}{8}u^8 + C$$

$$= \frac{1}{8}\tan^8 x + \frac{1}{3}\tan^6 x + \frac{1}{4}\tan^4 x + C$$