Anonym kod	LMA515 Matematik del B 170112	Sidnr	Poäng
		1	

1. Till nedanstående uppgifter skall korta lösningar redovisas, samt svar anges, på anvisad plats (endast lösningar och svar på detta blad, och på anvisad plats, beaktas).

(a) Bestäm $a > 0$ så att arean under grafen $f(x) = 1/(1+x)^2$ mellan $x = 0$ och $x = a$ är dubbelt så stor som arean mellan $x = a$ och $x = 2a$. (4p)

Lösning:

$$\int_0^a \frac{1}{(1+x)^2}dx = 2\int_a^{2a} \frac{1}{(1+x)^2}dx$$

$$\left[-\frac{1}{1+x}\right]_0^a = 2\left[-\frac{1}{1+x}\right]_a^{2a}$$

$$1-\frac{1}{1+a} = 2\left(\frac{1}{1+a}-\frac{1}{1+2a}\right)$$

$$\frac{a}{1+a} = \frac{2a}{(1+a)(1+2a)} \qquad 1+2a = 2$$

Svar: $a = 1/2$

(b) Bestäm inflexionspunkter till funktionen $f(x) = x^2 - \dfrac{8}{x}$. Ange intervall där funktionen är uppåt resp nedåt konkav. (dvs konvex/konkav) (3p)

Lösning:

$$f' = 2x + 8x^{-2} \qquad f'' = 2 - 16x^{-3}$$

$$= \frac{2(x^3-8)}{x^3} \qquad f''=0 \Rightarrow x = 2$$

x		0		2	
f''	+		−		+

Svar: Konvex i $x<0$ resp $x \geq 2$ Konkav: $0<x\leq 2$

(c) Ange den primitiva funktion till $f(x) = \left(x + \dfrac{1}{x}\right)^2$ som uppfyller $F(1) = 2$. (3p)

Lösning:

$$f(x) = x^2 + 2 + \frac{1}{x^2} \qquad F'(x) = f(x)$$

$$F(x) = \frac{x^3}{3} + 2x - \frac{1}{x} + C$$

$$\frac{1}{3} + 2 - 1 + C = 2 \qquad C = \frac{2}{3}$$

Svar: $F(x) = \dfrac{x^3+2}{3} + 2x - \dfrac{1}{x}$

Var god vänd!

(d) Beräkna $\int \sin(2x)(1+\cos(x))\,dx$. (3p)

Lösning:

$$\int 2\sin x\cos x(1+\cos x)\,dx = \left\{\begin{matrix} \cos x = t \\ -\sin x\,dx = dt \end{matrix}\right\}$$

$$= -2\int (t+t^2)\,dt = -2\left(\frac{t^2}{2}+\frac{t^3}{3}\right)+C$$

Svar: $-\cos^2 x - \frac{2}{3}\cos^3 x + C$

(e) Lös differentialekvationen $y'' + 4y' - 12y = 3 + e^{-2x}$. (3p)

Lösning:

$r^2+4r-12=0 \quad r=-2\pm\sqrt{4+12} = \left\{\begin{matrix}-6\\2\end{matrix}\right.$

$y_h = C_1e^{2x}+C_2e^{-6x}$

$y_{p_1} = A \Rightarrow -12A = 3 \quad A = -\frac{1}{4}$

$y_{p_2} = Be^{-2x} \Rightarrow 4B - 8B - 12B = 1 \quad B = -\frac{1}{16}$

Svar: $y = C_1e^{2x} + C_2e^{-6x} - \frac{1}{16}e^{-2x} - \frac{1}{4}$

(2) $x \neq -2 \qquad x=-2$ lodrät asymptot

$$\frac{f(x)}{x} = \frac{1}{x} - 2 + \frac{1}{2+x} \to -2 = k, \quad x \to \pm\infty$$

$$f(x) - kx = 1 + \frac{x}{2+x} \to 2 = m, \quad x \to \pm\infty$$

$y = -2x + 2$ sned asymptot

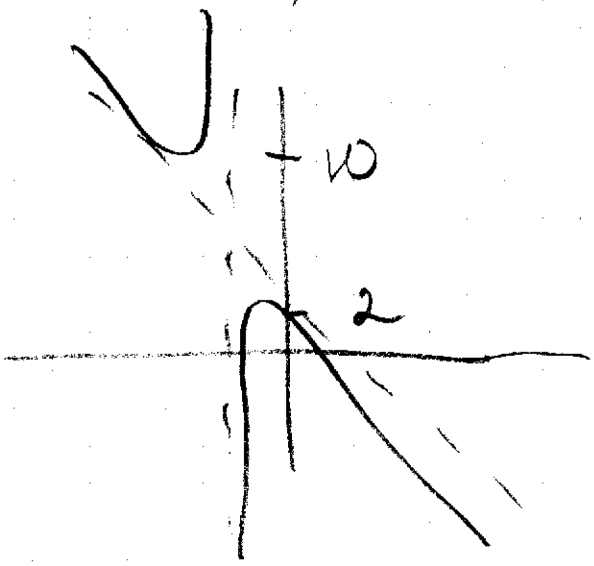
$$f'(x) = -2 + \frac{1\cdot(2+x) - x\cdot 1}{(2+x)^2}$$

$$= -2\,\frac{(x+2-1)(x+2+1)}{(2+x)^2}$$

x		-3		-2		-1	
f'	$-$		$+$		$+$		$-$

x	y
-3	10
-1	2

$x=-3$ lok min, $x=-1$ lok max



(3) $$\int_0^\infty \frac{1}{1+2e^x}\,dx = \left\{ \begin{matrix} e^x = t \\ x = \ln t \end{matrix} \quad dx = \frac{dt}{t} \right\}$$

$$= \int_1^\infty \frac{1}{t(1+2t)}\,dt = \int_1^\infty \frac{1}{t} - \frac{2}{1+2t} = \left[\ln\frac{t}{1+2t}\right]_1^\infty$$

$$= \ln\frac{1}{2} - \ln\frac{1}{3} = \ln\frac{3}{2}$$

(4) $f' = \dfrac{2x(2x-1) - x^2 \cdot 2}{(2x-1)^2}$

$= \dfrac{2x^2 - 2x}{(2x-1)^2} = \dfrac{2(x-1)\cdot x}{(2x-1)^2}$

$$\begin{array}{cccc} & 1/2 & 1 & x \\ f' & & - & + \end{array}$$

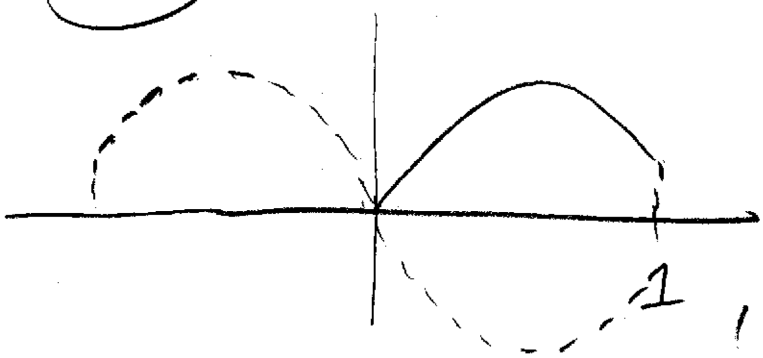
$x \geq 1 \Rightarrow$ växande $\Rightarrow$ invers finns

$y = \dfrac{x^2}{2x-1} \qquad x^2 - 2yx + y = 0$

$x = y \pm \sqrt{y^2 - y}$

$x \geq 1 \Rightarrow f^{-1}(y) = y + \sqrt{y^2 - y}$

(5)



rot kring x: $V = \int_0^1 \pi y^2 dx = \int_0^1 \pi x^2(4-3x) dx$

$= \pi \left[\dfrac{4x^3}{3} - \dfrac{3x^4}{4}\right]_0^1 = \dfrac{7\pi}{12}$

y: $V = \int_0^1 2\pi x y dx = \int_0^1 2\pi x^2 \sqrt{4-3x}\, dx$

(5) $= \left\{ 4 - 3x = t^2 \right\} =$

$$= \frac{2\pi}{} \int_1^2 \left(\frac{4 - t^2}{3} \right)^2 \frac{2}{3} t^2 \, dt$$

$$= \frac{4\pi}{27} \left[16 \frac{t^3}{3} - \frac{8t^4}{4} + \frac{t^7}{7} \right]_1^2$$

(6) $f' - \frac{2}{x} f = \frac{x-1}{x}$

$$e^{\int -\frac{2}{x} dx} = e^{-2\ln|x|} = x^{-2}$$

$$D\left(x^{-2} f\right) = \frac{x-1}{x^3}$$

$$x^{-2} f = \int \frac{x-1}{x^3} dx + C$$

$$= + \frac{1}{2x^2} - \frac{1}{x} + C$$

$$2 = \frac{1}{2} - 1 + C \qquad C = \frac{5}{2}$$

$$f(x) = \frac{1}{2} - x + \frac{5x^2}{2}$$

(7) i $\int \frac{\sin^2 x}{1 + \frac{\sin^2 x}{\cos^2 x}} = \int \cos^2 x \sin^2 x$

$$= \int \frac{\sin^2 2x}{4} = \int \frac{1 - \cos 4x}{8}$$

7 ii $\displaystyle\int \frac{x^2}{x^3+x^{-3}} = \int \frac{x^5}{x^6+1}\,dx$

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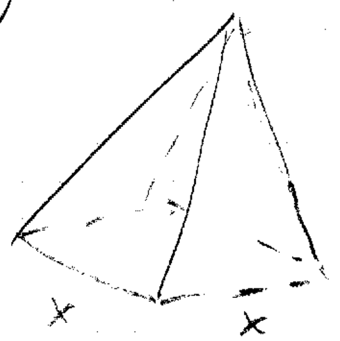

$$V = \frac{x^2 h}{3}$$

$$A = x^2 + 4 \cdot \frac{x \cdot \sqrt{h^2 - \left(\frac{x}{2}\right)^2}}{2}$$

$$= x^2 + 2x\sqrt{\left(\frac{3V}{x^2}\right)^2 - \frac{x^2}{4}}$$

$$\frac{dA}{dx} = 0$$

9

$$v' = g - kv^2$$

$$\frac{dv}{g - kv^2} = dt$$

$$\frac{dv}{\alpha^2 - v^2} = g\,dt \qquad \alpha = \sqrt{\frac{g}{k}}$$

$$\int \left(\frac{1}{\alpha - v} + \frac{1}{\alpha + v}\right) dv = \int 2\alpha g\,dt + C$$