Svar till MVE016

april 2013

(1) a) $$\int_1^e x^3 \ln x \, dx = \left[\frac{x^4}{4} \ln x \right]_1^e - \int_1^e \frac{x^4}{4} \cdot \frac{1}{x} \, dx =$$

$$= \frac{3e^4+1}{16}$$

b) $$\int_0^2 \frac{dx}{x^2+4} = \frac{1}{2}\left[\arctan \frac{x}{2} \right]_0^2 = \frac{\pi}{8}$$

c) $$\int \sin^3 x \, dx = \int \sin x \cdot (1-\cos^2 x) \, dx =$$

$$= -\cos x + \frac{\cos^3 x}{3} + C$$

(2) a) Nej, termerna går ei mot 0

b) Ja, använd kvotkriteriet

(3) $y_h = A\sin 2x + B\cos 2x$

y_p: Ansätt $y_p = p\sin x + q\cos x$

Man får $y_p = \sin x$

Begynnelsevillkoren ger $y = \sin x + \sin 2x$

(4) Taylorutveckla:

$$\ln(1+\sin x^2) = x^2 - \frac{x^4}{2} + O(x^6)$$

$$\cos(x \arctan 2x) = 1 - 2x^4 + O(x^6)$$

Så $f(x) = 1 + x^2 - \frac{5x^4}{2} + O(x^6)$

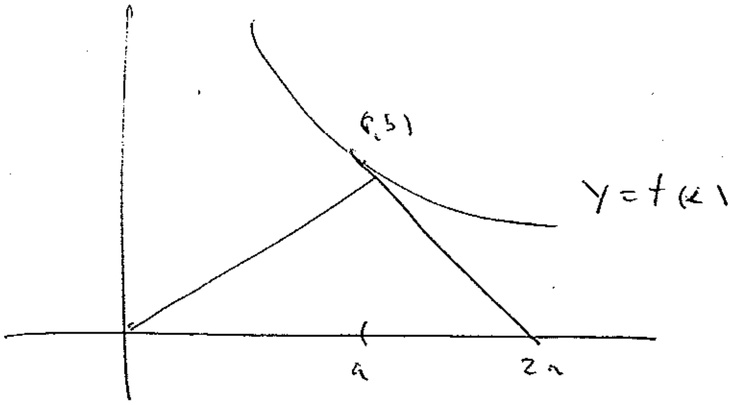
och $f^{(IV)}(0) = -60$

(5) $$\frac{x}{x^2+1} - \frac{c}{3x+1} = \frac{(3-c)x^2 + x - c}{(x^2+1)(3x+1)}$$

c måste vara $= 3$ för

att integralen skall bli konvergent

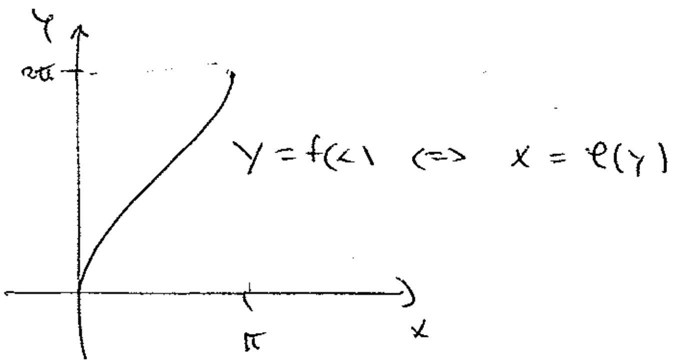
(6)



Differentialekvation : $y' = -\frac{y}{x}$

med lösningar $y = \frac{c}{x}$ $(c > 0)$

(7)



$$\int_0^{\pi} f(x)\,dx + \int_0^{2\pi} g(y)\,dy = \pi \cdot 2\pi \Rightarrow \int_0^{2\pi} g(y)\,dy = \pi^2 - 2$$

(8) a) se boken

b) $$\sum_{n=1}^{\infty} e^{1-xn} = e^{1-x}\left[1 + e^{-x} + e^{-2x} + \dots\right]$$

$$= e^{1-x}\,\frac{1}{1-e^{-1}} = \frac{e^{2-x}}{e-1} \quad \text{om } |e^{-x}| < 1$$

dvs $x > 0$