Mat Analys MVE016

Tentamen 30/8 -13

Svar / Lösningar

1 a) $$\int_0^1 \frac{dx}{\sqrt{2x+1}} = \sqrt{3} - 1$$

b) $$\int x^2 \arctan x = \left[\frac{x^3}{3} \arctan x \right] - \int \frac{x^3}{3} \frac{1}{x^2+1}\, dx =$$

$$= \frac{x^3}{3} \arctan x - \frac{1}{3} \int x - \frac{x}{x^2+1}\, dx =$$

$$= \frac{x^3}{3} \arctan x - \frac{x^2}{6} + \frac{1}{6} \ln(x^2+1) + C$$

c) $$\int_0^{\pi/4} \frac{dt}{\cos t} = \int_0^{\pi/4} \frac{\cos t\, dt}{1 - \sin^2 t} = \int_0^{1/\sqrt{2}} \frac{ds}{1 - s^2} =$$

$$= \frac{1}{2} \left[\ln \frac{1+s}{1-s} \right]_0^{1/\sqrt{2}} = \frac{1}{2} \ln \frac{\sqrt{2}+1}{\sqrt{2}-1} =$$

$$= \ln(\sqrt{2}+1)$$

2

a) Konvergent enligt kvotkriteriet

b) Geometrisk serie, konvergent om $|-4x|<1$

dvs $|x|<\frac{1}{4}$

3

$$y_h = Ae^{-x} + Be^{-2x}$$

$$y_p = e^{x}$$

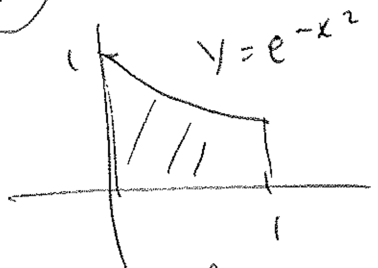
$$y = 2e^{-2x} - 3e^{-x} + e^{x}$$

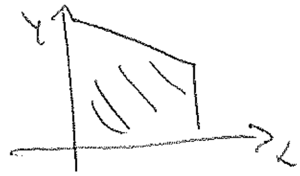
4

$$\frac{\ln(1+2x) - 2\ln(1+x)}{1-\cos(2x)} = \frac{2x - \frac{(2x)^2}{2} - 2\left(x - \frac{x^2}{2}\right) + O(x^3)}{1 - \left[1 - \frac{(2x)^2}{2}\right] + O(x^3)}$$

$$= \frac{-x^2 + O(x^3)}{2x^2 + O(x^3)} \to -\frac{1}{2} \quad \text{då} \quad x \to 0$$

5





$y = e^{-x^2}$ konvex så

$$\int_0^1 e^{-x^2}\,dx \le 1\cdot\frac{1+\frac{1}{e}}{2} = \frac{1+e}{2e}$$

(6) $yy' = \frac{\ln x}{x}$ separabel:

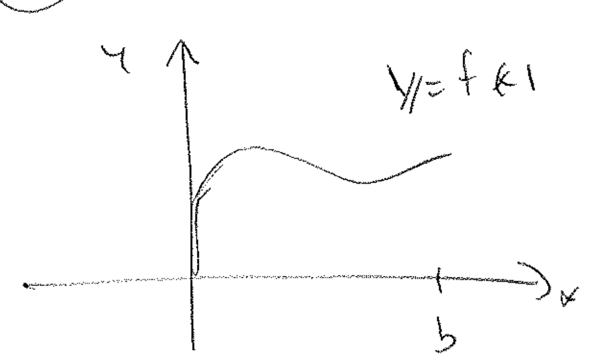
$y\,dy = \frac{\ln x}{x}\,dx$

$\frac{y^2}{2} = \frac{(\ln x)^2}{2} + C$, $y^2 = (\ln x)^2 + C_1$

$y(1) = 2 \Rightarrow y^2 = (\ln x)^2 + 4$

och $y > 0$ så $y = \sqrt{4 + (\ln x)^2}$

(7)



$\pi \int_0^b f(x)^2\,dx = b^2$ Derivering m.a.p b ger

$\pi f(b)^2 = 2b$ dvs $f(b) = \sqrt{\frac{2b}{\pi}}$

8b

$$VL = x\int_0^x f(v)\,dv - \int_0^x f(v)\cdot v\,dv$$

Derivatan map x blir =

$$= \int_0^x f(v)\,dv + x\,f(x) - x\,f(x) = \int_0^x f(v)\,dv$$

$$HL = \int_0^x \left[\int_0^v f(t)\,dt\right] dv$$

Derivatan map x blir =

$$= \int_0^x f(t)\,dt$$

Och Eftersom bägge led $= 0$ då $x = 0$

följer påståendet