Matematisk analys I1 MVE016

Svar/lösn till tentamen 13/4 - 2015

1)

a) $\int x^2 \cdot \ln(x^2)\,dx = \int 2x^3 \cdot \ln x\,dx = \left(\frac{x^4}{2}\ln x\right) -$

$- \int \frac{x^4}{2}\cdot\frac{1}{x}\,dx = \frac{x^4}{2}\ln x - \frac{x^4}{8} + C$

b) $\int_0^1 x^2 e^{x^3+1} = \left[\frac{e}{3}e^{x^3}\right]_0^1 = \frac{e^2}{3} - \frac{e}{3}$

c) $\int_1^2 \frac{dx}{\sqrt{x}\,(1+4x)} = \left[\begin{array}{l} x = t^2 \\ dx = 2t\,dt \end{array}\right] = \int_1^{\sqrt{2}} \frac{2t\,dt}{t(1+4t^2)} =$

$= \int_1^{\sqrt{2}} \frac{2\,dt}{1+4t^2} = \left[\arctan 2t\right]_1^{\sqrt{2}} =$

$= \arctan 2\cdot\sqrt{2} - \arctan 2$

2 a)

Kvotkriteriet : $\dfrac{a_{n+1}}{a_n} = \dfrac{[(n+1)!]^2 \cdot (2n)!}{(n!)^2 \cdot (2n+2)!} =$

$= \dfrac{(n+1)^2}{(2n+1)(2n+2)} \to \dfrac{1}{4}$ så $\sum_1^\infty a_n$ är konvergent

b)

$a_n = \dfrac{2^n}{n}$ och $\dfrac{a_{n+1}}{a_n} = \dfrac{2n}{(n+1)} \to 2$ då $n \to \infty$

så $R = \dfrac{1}{2}$ $\sum_1^\infty a_n (x-3)^n$ konvergent

om $|x-3| < \dfrac{1}{2}$

För $x = \dfrac{7}{2}$ ~~och~~ $\cancel{x = \frac{5}{2}}$ blir serien

$\sum_1^\infty \dfrac{1}{n}$ alltså divergent

om $x = \dfrac{5}{2}$ får vi $\sum_1^\infty \dfrac{(-1)^n}{n}$ som är konvergent

Svar: Serien konvergent om $\dfrac{5}{2} \leq x < \dfrac{7}{2}$

(3) $$e^{-2x} = 1 - 2x + \frac{(2x)^2}{2} + O(x^3)$$

$$e^{-\frac{x}{2}} = 1 - \frac{x}{2} + \frac{(x/2)^2}{2} + O(x^3)$$

så $$\frac{e^{-2x} - 4e^{-\frac{x}{2}} + 3}{2x^2} = \frac{\frac{3}{2}x^2 + O(x^3)}{2x^2} \to \frac{3}{4} \text{ då } x \to 0$$

(4) $$9y'' + y = 3x + e^{-x}$$

Homogenlsg: $Y_h = A\cos\frac{x}{3} + B\sin\frac{x}{3}$

Partikulärlsg: $Y_p = 3x + \frac{e^{-x}}{10}$

Svar: $$Y = A\cos\frac{x}{3} + B\sin\frac{x}{3} + 3x + \frac{e^{-x}}{10}$$

(5) $$\int_0^\infty \frac{dx}{e^{x+1} + e^{3-x}} = \left[\begin{matrix} e^x = t \\ dx = \frac{dt}{t} \end{matrix}\right] = \int_1^\infty \frac{dt}{t(e \cdot t + e^3 \cdot \frac{1}{t})} = \int_1^\infty \frac{dt}{e(t^2 + e^2)} =$$

$$= \left[\frac{1}{e} \cdot \frac{1}{e} \arctan\frac{t}{e}\right]_1^\infty = \frac{1}{e^2}\left(\frac{\pi}{2} - \arctan\frac{1}{e}\right)$$

(6)

$$\cos x^2 = 1 - \frac{x^4}{2} + \frac{x^8}{24}\cos\theta x^2$$

så $\displaystyle\int_0^1 \cos x^2\,dx = \int_0^1 1 - \frac{x^4}{2}\,dx + \int_0^1 \frac{x^8}{24}\cos\theta x^2\,dx$

där $\displaystyle\left|\int_0^1 \frac{x^8}{24}\cos\theta x^2\,dx\right| \le \int_0^1 \frac{x^8}{24}\,dx = \frac{1}{216} < 0{,}05$

så $\displaystyle\int_0^1 \cos x^2\,dx \approx \int_0^1 1 - \frac{x^4}{2}\,dx = \frac{9}{10}$

(7)

Sätt $x=0$ så får vi $f(0)=1$

Derivering ger

$f'(x) + 2x = 4x\,f(x)$

Vi skall alltså lösa $y' - 4xy = -2x$

Man får $\left(ye^{-2x^2}\right)' = -2xe^{-2x^2} \Rightarrow$

$\displaystyle ye^{-2x^2} = \frac{e^{-2x^2}}{2} + C \Rightarrow y = \frac{1}{2} + Ce^{2x^2}$

$f(0)=1 \Rightarrow C = \frac{1}{2}$ så $\displaystyle y = \frac{1+e^{2x^2}}{2}$

(8) Sätt $f(x) = \left| \sin|\pi - x| - \cos x \right|$

Då är $f(x) = \begin{cases} \cos x - \sin x & 0 \le x \le \pi/4 \\ \sin x - \cos x & \pi/4 \le x \le \pi \\ -\sin x - \cos x & \pi \le x \le \frac{7\pi}{4} \\ \sin x + \cos x & 7\pi/4 \le x \le 2\pi \end{cases}$

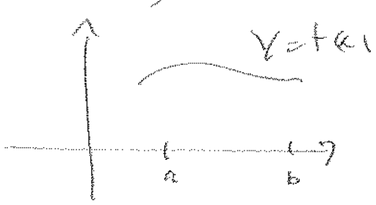
och $\displaystyle\int_0^{2\pi} f(x)\,dx = \Big[\sin x + \cos x\Big]_0^{\pi/4} + \Big[-\cos x - \sin x\Big]_{\pi/4}^{\pi} +$

$\displaystyle + \Big[\cos x - \sin x\Big]_{\pi}^{7\pi/4} + \Big[-\cos x + \sin x\Big]_{7\pi/4}^{2\pi} =$

$= \sqrt{2} - 1 + 1 + \sqrt{2} + \sqrt{2} + 1 - 1 + \sqrt{2} = 4 \cdot \sqrt{2}$

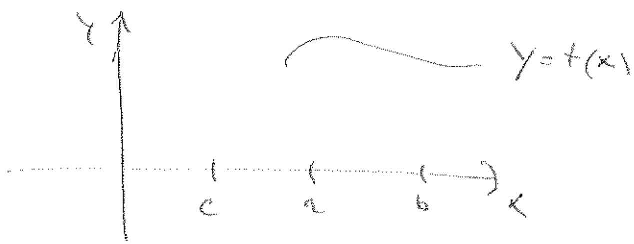
(9)

a)



$V = 2\pi \displaystyle\int_a^b f(x)\, x\, dx$

b)



$V = 2\pi \displaystyle\int_a^b f(x)(x - c)\, dx$

Härledningen är densamma som för den vanliga rotationen