Sufficient conditions for isomorphisms between function algebras

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Norm-multiplicative maps

Let $A \subset C(X)$ be a function algebra on a loc. compact Hausdorff space X, i.e. A is uniformly closed and strongly separates the points of X.

We assume that X contains the Shilov boundary ∂A of A, and together - its Choquet boundary δA.

By $\|f\| = \max_{x \in X} |f(x)|$ we denote the *uniform norm* of $f \in A$.

Theorem 1. [T, 2009]

Let $T \colon A \to B$ be a surjective (in general not linear) map between two function algebras A and B. If

$$\|Tf\, Tg\| = \|fg\|$$

for all $f, g \in A$ then there is a homeomorphism $\psi \colon \delta B \to \delta A$ so that

$$|(Tf)(y)| = |f(\psi(y))|$$

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The theorem holds also if A, B are (uniformly) dense subalgebras of function algebras such that $\delta\overline{A} = p(A),\ \delta\overline{B} = p(B)$ where $p(A),\ p(B)$ are the sets of p-points, i.e. the strong boundary points of A, B.

In particular, it holds for semisimple algebras A with $\delta\overline{A} = p(A)$ by the way of the Gelfand algebras $\widehat{A}$.

Corollary 1.

If $T : A \to B$ is a surjective multiplicative map between two function algebras A and B which preserves the norms, i.e.

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Indeed, in this case $\|Tf\ Tg\| = \|T(fg)\| = \|fg\|$ and the result follows directly from Theorem 1.

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In general, the equality $|(Tf)(y)| = |f(\psi(y))|$ does not necessarily imply that T is a composition (or, weighted composition) operator, nor that it is an algebra isomorphism.

An immediate counterexample is the conjugacy map $T : f \mapsto \overline{f}$.

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Spectral multiplicative conditions

Given an $f \in A$, we denote by $\sigma_\pi(f)$ the *peripheral spectrum* of f, i.e. $\sigma_\pi(f)$ is the set of values of f with maximum modulus, namely,

$$\sigma_\pi(f) = \{f(x) \colon |f(x)| = \|f\|,\ x \in X\}.$$

Clearly, $\sigma_\pi(f) \subset \{z \in \mathbb{C} \colon |z| = \|f\|\}$.

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If $T : A \to B$ is a surjective map between function algebras such that

$$\sigma_\pi(Tf\,Tg) \subset \sigma_\pi(fg)\ [\text{or, } \sigma_\pi(fg) \subset \sigma_\pi(Tf\,Tg)]$$

for all $f, g \in A$, then there is a function $\alpha \in C(\delta B)$ with $\alpha^2 = 1$ so that

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As a consequence we see that there is a clopen set $E \subset \delta B$ so that $Tf = f$ on E, and $Tf = -f$ on $\delta B \setminus E$.

Clearly, in this case, the operator $\alpha T = f \circ \psi$ is an algebraic isomorphism.

The condition $\sigma_\pi(Tf\,Tg) \cap \sigma_\pi(fg)) \neq \varnothing$, $f, g \in A$ is a more general than $\sigma_\pi(Tf\,Tg) \subset \sigma_\pi(fg)$.

However, it alone does not imply that T a composition (or, weighted composition) operator, unless X is a metric space.

Theorem 3. [Johnson-T, 2013]

If a surjection $T: A \to B$ between function algebras $A \subset C(X)$, $B \subset C(Y)$ on metric spaces X, Y is such that

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It is not known if Theorem 3 holds for non-metric spaces without additional conditions.

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If a surjection $T : A \to B$ between function algebras is such that

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and $\sigma_\pi(Tf)$ is a singleton whenever σ_π is a singleton, then

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Note that if, in addition $dist(\sigma_\pi(Tf), \sigma_\pi(f)) < 2$, then $\alpha = 1$, i.e. $(Tf)(y) = f(\psi(y))$ in all previous theorems.
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If a surjection $T: A \to B$ between function algebras is such that

$$\sigma_\pi(Tf\,Tg) \cap \sigma_\pi(fg) \neq \varnothing, \ \ f, g \in A$$

and $\sigma_\pi(Tf)$ is a singleton whenever σ_π is a singleton, then

$$(Tf)(y) = \alpha(y)\, f(\psi(y))$$

for all $y \in \delta B$ and $f \in A$, where $\alpha \in C(\delta B)$ with $\alpha^2 = 1$.

Note that if, in addition $dist(\sigma_\pi(Tf), \sigma_\pi(f)) < 2$, then $\alpha = 1$, i.e. $(Tf)(y) = f(\psi(y))$ in all previous theorems.
Therefore, T is a composition operator on δB, and consequently, an isometric algebra isomorphism.

Theorem 5. [Johnson-T, 2013]

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All previous theorems hold also in the case when the uniform closures $\overline{A},\ \overline{B}$ are function algebras with $\delta\overline{A} = p(A),\ \delta\overline{B} = p(B)$.

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For instance, Theorem 3 holds for any Lipschitz algebra.

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Corollaries

As direct corollaries of the above theorems we obtain:

Corollary 2.

If a surjective multiplicative map $T: A \to B$ between two function algebras is such that

$$\sigma_\pi(Tf) \subset \sigma_\pi(f)$$

for all $f \in A$, then there is a function $\alpha \in C(\delta B)$ with $\alpha^2 = 1$ so that

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If a multiplicative surjection $T\colon A \to B$ between function algebras $A \subset C(X),\ B \subset C(Y)$ on metric spaces X, Y is such that

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Almost isomorphisms [to appear in the PAMS]

Theorem 6.

$A \subset C(X)$, $B \subset C(Y)$ – function algebras. $T\colon A \to B$ – surjective map with $\|Tf\,Tg\| = \|fg\|$, $f, g \in A$, such that there is an ϵ, $0 \le \epsilon < 2/3$, so that

$$\sigma_\pi(Tf\,Tg) \subset \mathcal{O}_{\epsilon\,\|fg\|}(\sigma_\pi(fg))$$

for all $f, g \in A$, $\|f\| = 1$.
Then there is a homeomorphism $\psi\colon \delta B \to \delta A$ and a continuous function $\alpha\colon \delta B \to \{\pm 1\}$ such that

$$|(Tf)(y) - \alpha(y)\, f(\psi(y))| \le 2\epsilon\, |f(\psi(y))|,$$

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Hence T is an almost weighted composition operator.

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Note that if $f(\psi(y)) \neq 0$, the condition in Theorem 6 can be rewritten as

$$\left| \frac{(Tf)(y)}{\alpha(y)\, f(\psi(y))} - 1 \right| \leq 2\epsilon.$$

Analogous to Theorem 6 result holds if the hypothesis is replaced by its symmetric $\sigma_\pi(fg) \subset \mathcal{O}_{\epsilon\,\|fg\|}(\sigma_\pi(Tf\,Tg))$, $f, g \in A$.

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Then there is a homeomorphism $\psi\colon \delta B \to \delta A$ so that

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