William George Bade

(29 May 1924 – 10 August 2012)

H. G. Dales, Lancaster

Banach algebras - Gothenburg

31 July 2013

Reference

Obituary: William George Bade, 1924–2012, *Bulletin of the London Mathematical Society*, 45 (2013), 875–888.

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William George Bade

Bill Bade was:

• the instigator in 1974 (with Phil Curtis) of this sequence of conferences on **Banach algebras**;

• a founding father (again with Phil Curtis) of **automatic continuity theory** for Banach algebras and a major contributor to our subject;

• the thesis adviser to 24 successful PhD students at Berkeley, many of whom have also made significant contributions;

• a pillar of the Berkeley maths department, especially Vice-Chair for Graduate students, for about 40 years;

• with his wife Elly, and family, a wonderfully generous host in California;

• of great kindness, overwhelming integrity, and sound judgement.

Father: William Frederic Badè

William Frederic Badè was born in 1871; he was a farmer's boy in rural Minnesota, the son of emigrants from Germany.

He became a distinguished (Moravian) minister, linguist, theologian, and archeologist.

Professor of *Old Testament Literature and Semitic Languages* at the Pacific School of Religion in Berkeley from 1902.

He married Evelyn Marianne Ratcliff in 1906; she died from complications associated with childbirth in 1907.

He was a close friend of the iconic naturalist, conservationist, and explorer **John Muir**, who founded the **Sierra Club** in 1892; he was a moutaineer in the Sierras.

Author of the then controversial volume: *The Old Testament in the light of to-day. A study in moral development* of 1915.

William Frederic Badè – continued

WFB was the biographer of John Muir: he wrote *The life and letters of John Muir, Volumes 1 and 2*, Houghton Mifflin, Boston and New York, 1923.

WFB was a seminal figure in the development of the archaeological exploration of the Near East, and wrote books on this.

In 1935, all the family accompanied him on the final excavation season at Tell en-Nasbeh; the trip was by steamer ship, west across the Pacific via Japan and the far East, returning via Germany.

At age 11, Bill learned to speak some 'street Arabic'.

The archaeological work of WFB is commemorated in the *Badè Museum* at the Pacific School of Religion, Berkeley.

Maternal grandfather

Bill's maternal grandfather was **George White Marston** (1850–1946), a very successful entrepreneur (eventually with a large Department store) and wealthy philanthropist from San Diego.

He became a founding trustee of Pomona College in 1887, and a member of the Pomona College Board from then until his death; he was also a very generous donor to the College, which named its central quadrangle in his honour.

With others he bought land in California, and gave it to the State; it is now the Anza-Borrego Desert State park, the largest state park in California.

There is a two-volume biography of George Marston.

Maternal grandmother

Bill's maternal grandmother was **Anna Lee Gunn** (1853–1940), whose own mother had travelled to California from Pennsylvania 'round Cape Horn' over six months in 1851 with two small children; Anna's father had travelled earlier to Ca. by crossing Mexico on a horse.

We have the diary of that epic voyage.

It seems that Anna Lee Gunn was the first white child to be born in Sonoma County, California.

Mother

Bill's mother was **Elizabeth Le Breton Marston**, the third child (of five) of George and Anna Marston.

She was born in San Diego in 1884, and graduated from Wellesley College in 1905 as a joint German and Zoology major.

Her 'European tour' to France and England was from June to October, 1914.

She married W. F. Badè in January 1917.
After their marriage, Bill's parents spent the next six months on a trip which included following John Muir's footsteps on his famous 'Thousand Mile Walk to the Gulf'.

Bill was born in Oakland, California, on
29 May, 1924. He had an older sister, Betsy.

Schooling

Bill was the only member of the Berkeley Mathematics Faculty who was a schoolboy in the city - and two grandchildren went to the same elementary school.

Bill's father died in March 1936, when Bill was a 11-year old boarder at Ojai school in rural southern California.

In 1937, he moved with his mother and older sister to his grandfather's house in San Diego.

He attended San Diego High School from 1937 to 1942.

Bill's mother died in 1987, just short of her 103^{rd} birthday, and so she was a widow for more than 50 years.

Because of his family connection, Bill chose to enter Pomona College as a freshman in 1942.

Pomona

It is strange to see the elementary nature of the courses that Bill took at Pomona College; there were beginning courses in German, and the mathematics consisted of 'college algebra, trigonometry, and an introduction to analytic geometry'.

But fellow students included **Kenneth Cooke**, **Robert Vaught**, and **Victor Klee**, who all became famous mathematicians.

Caltech

In the spring of 1943, the US Navy sent a recruitment team to Pomona; Bill volunteered, and enlisted in the 'V-12 program'.

In September, 1943, Bill was sent to Caltech = **California Institute of Technology** to study Electrical Engineering; the Navy had decided to train him to become a radar officer.

In the event, Bill majored in physics, completing three years' of classes in two years. His mathematics courses included 'Advanced calculus' and 'Advanced differential equations' and not much more mathematics.

He graduated with a BS on 22 June, 1945.

Navy Service

The war in Europe had just finished in June 1945, and soon the dropping of the atomic bombs in Japan led to the end of the war in Asia.

Thus Bill did not see any fighting. But he still had to serve the Navy: he was sent to Columbia University in New York, and then to Harvard Business School, to train to be a Disbursing Officer, and then he served as such in the Pacific, mainly on the atoll of Truk.

Bill was discharged from active duty in June, 1946.

UCLA

Bill enrolled at UCLA in 1946 to study physics, but he soon became more interested in mathematics, and switched to this subject after one semester, receiving his MA in 1948.

Bill then became a PhD student of **Angus Taylor**, who had become a full professor of mathematics at UCLA in 1944.

Taylor was the author of the seminal text on functional analysis that influenced generations.

Taylor's own work focused on the spectral theory of linear operators - and Bill wrote his thesis on this topic.

Bill's PhD was conferred in 1951; **Richard Arens** was on the Committee.

Taylor was also a substantial mountaineer in the Sierras (and the Alps).

Bill's comments

Bill wrote:

'In 1947 I was a beginning graduate student in mathematics at UCLA with no idea what direction I was going. ... I found an expository article of Angus ... His aim was to tell how mathematicians find research problems and, as we say "do mathematics" .... This article was so persuasive and exciting that I knew I had found my niche and I am still there. I enrolled on his course on Banach spaces and soon I was one of his thesis students. He was a very clear and careful teacher and a wise thesis adviser, encouraging but not directing, so the student had the joy of discovery.'

Stillwater, Oklahoma

Three hot weeks in June and July, 1950:

A symposium was sponsored by the US Office of Naval Research; participants, including Taylor, were invited to bring their graduate students, and so Bill accompanied Taylor on the long train journey to Oklahoma.

The graduate students earned their expenses by writing up the lecture notes of the distinguished speakers; Bill was allocated to **Nelson Dunford** of Yale, and thereby his career was launched - see later.

Berkeley and the Loyalty Oath

Bill became an instructor at Berkeley for 1951–52, at the recommendation of Taylor.

This was just after the 'loyalty oath controversy' racked the University of California, including the Department of Mathematics at Berkeley, where **John Leroy Kelley** (author of 'General Topology') was a significant 'non-signer' and was dismissed in 1950.

Bill had to sign.

The California Supreme Court held the 'disclaimer oath' to be unconstitutional in 1952, and the non-signers were reinstated; Kelley played a very significant role in the later history of the UCB Mathematics Department.

See the book of Calvin Moore on this.

(Frank Bonsall left Oklahoma at the same time on the same issue.)

Marriage

Eleanor (Elly) Jane Barry, of La Cañada, near Los Angeles, was a student at Pomona College from 1944 to 1948, majoring in history.

She and Bill met in 1948. For the year 1951–52, Elly moved to San Francisco as a school librarian; she and Bill were married on 2 July, 1952.

To anticipate: they celebrated a joyous diamond wedding in 2012 surrounded by 6 children, etc.

See the pictures.

The years at Yale

Nelson Dunford was a faculty member at Yale; **Jacob Schwartz** obtained a PhD under Dunford in 1951, and together they set out to write 'Linear operators', which was to become one of the most impressive and influential books in functional analysis of the era.

They were supported during the eight years of the writing of Volume I by the Office of Naval Research; these funds allowed the appointment of two research assistants for the three years 1952–55: Bill and **Robert Bartle.**

The Chairman at Yale was **G. Hedlund**; faculty members included **Einar Hille**, **Charles Rickart** (the author of the definitive monograph on Banach algebras), **Nathan Jacobson**, **Shizou Kakutani**, and **Dunford** and **Schwartz**. **Henry Helson** and **John Wermer** were Instructors, and **Phil Curtis** was a second-year graduate student of Rickart, and so it was that Bill and Phil met at Yale.

The social scene

Elly has very happy memories of the three years in Yale; she says it was a very friendly place, with many 'picnics, parties, outings, and dances', where faculty and graduate students mixed easily.

Maybe it was more relaxed than it would be today.

Their two oldest children were born in New Haven; **Michael** in 1953 and **Sarah** in 1954.

Appointment at Berkeley

The year 1955 was the beginning of a great period of expansion of the Mathematics Department at Berkeley

A member of the Department, **Frank Wolf**, came to Yale in 1955, and proposed that both **Henry Helson** and Bill become Assistant Professors at Berkeley; Bill and Henry started employment on the same day in 1955, and they remained firm friends and close colleagues for more than 50 years.

Bill remained at Berkeley for the remainder of his career.

Also in 1955, **Phil Curtis** became an instructor at UCLA, in due course becoming a full Professor and Chairman (from 1971 to 1974).

Yale again

Bill and Phil obtained sabbatical leave for the year 1958–59. **Hille** at Yale had arranged that this year be a 'special year in functional analysis'.

Colleagues from Yale during this year of analysis included **Hille**, **Rickart**, **Dunford**, and **Schwartz**; visitors included **Walter Rudin**, **Edwin Hewitt**, and **John Wermer**.

This was the year when they wrote the two seminal papers on $C(X)$, to be discussed below.

Thus Bill and Elly took three little children, including **Alan** (1957), back to the east coast.

Graduate students at Berkeley

From about 1962 to 1990, Bill had a major role with respect to graduate students at Berkeley, latterly as Vice-Chair for Graduate Students. Basically he designed the programme.

From the mid-60s to the late 80s, Berkeley had the largest graduate school in mathematics in the USA, with around 400 such students enrolled, so this was a very influential position for the future of mathematics. In 1975, they awarded 44 PhDs in Mathematics: a record?

The Department's policy, strongly supported by Bill, was that, as part of a major public university, they should not be very selective in admitting graduate students, but should admit a broad range of applicants from a wide variety of institutions in the US.

Cal. Moore wrote: 'This was service of exceptional length and quality, going far beyond the call of duty.'

Bill's PhD students

Here is the official list:

1. 1959: Robert S. **Freeman**, 'Self adjoint boundary conditions for the Laplacian operator'.

2. 1961: Tamio **Ono**, 'Local theory in functional analysis'.

3. 1962: Sandra Barkdull **Cleveland**, 'Continuity of homomorphisms of noncommutative algebras'.

   Emeritus at Cal State Sacramento.

4. 1963: Galen Lathrop **Seever**, 'Measures on F-spaces'.

   Many years at the University of Texas; deceased 1991.

5. 1963: William Hall **Sills**, 'Arens multiplication and spectral theory'.

   Emeritus at San Jose State University.

6. 1966: Charles A. **Akemann**, 'The dual space of an operator algebra'.

   Professor at UC Santa Barbara.

7. 1966: Morisuke **Hasumi**, 'Invariant subspace theorems for finite Riemann surfaces'.

   Professor Emeritus, Ibaraki University, Mito, Japan.

8. 1967: James DeWitt **Stein**, 'Continuity of homomorphisms of von Neumann algebras'.

   Professor at California State Long Beach.

1967: Benjamin Baxter **Wells**, Jr., 'Uncomplemented subspaces of continuous functions and weak compactness of measures'.

Professor at University of Virginia

1968: Seymour Zane **Ditor**, 'Linear operators of averaging and extension'.

Professor at University of Western Ontario.

1971: John Edwin **Wolfe**, 'Injective Banach spaces of type $C(T)$'.

Professor at Oklahoma State University.

1973: Frederick Knowles **Dashiell**, Jr., 'Isomorphism problems for the Baire classes and strict convexifiability'.

1973: Alain Juan **Etcheberry**, 'Isomorphism of spaces of continuous functions'.

Retired from the University of Simon Bolivar, Caracas, Venezuela.

1976: Marc Phillip **Thomas**, 'The algebraic structure of a continuous linear operator on a Frechet space'.

Professor at California State University Bakersfield.

1977: Juan Camilo **Ochoa**, 'Automatic continuity in algebras of differentiable functions of several variables'.

1978: Boon-Hua **Ong**, 'Invariant Subspace Lattices for a Class of Operators'.

School of Mathematical Sciences, Universiti Sains Malaysia.

1979: Frederick Michael **Goodman**, 'Closed derivations in commutative C^* algebras'.

Emeritus professor at the University of Iowa.

1984: Ngo Nhu Phu **Viet**, 'Automatic continuity in algebras of differentiable functions'.

Professor at California State University, Long Beach.

1985: Elizabeth Jane **Strouse**, 'Embedding the algebra of formal power series in several variables into a Banach algebra'.

Professor at the University of Bordeaux, France.

1988: Peter Andrew **Detre**, 'Multipliers of weighted Lebesgue spaces'.

1991: Mladen **Despic**, 'Automorphisms of radical convolution algebras'.

1993: Branka **Pavlovic**, 'Automatic continuity of Lipschitz algebras'.

1993: Volker **Runde**, 'Problems in Automatic Continuity'.

Professor at the University of Alberta, Edmonton, Canada.

May 1994: Nikolai Isaac **Weaver**, 'Lipschitz Spaces'.

Professor at Washington University, St.Louis.

Visits to Europe

Bill and Phil Curtis were Guest Professors at Aarhus in 1969–70, at the invitation of **Svend Bundgaard**.

By now Bill and Elly had six children, with the arrival of **Anne** in 1960, **Edward** in 1962, and **Margaret** in 1965 to form a pleasing, alternating sequence.

What an heroic achievement to take six children to Europe in 1969! The family travelled by boat from San Francisco, through the Panama Canal, to Lisbon, and eventually to Southampton, from which they visited London before flying to Copenhagen.

The whole family was established in a house in Aarhus, and the oldest five children were enrolled in local Danish schools, where they learnt to speak Danish.

Meetings in Aarhus

That year, Bill and Elly met **Kjeld Bagger Laursen**, recently returned to Denmark from Minnesota with a wife, **Virginia**, and a PhD.

Bundgaard also managed to find some money to bring **Wiesław Żelazko** from Warsaw, a rare achievement in those days of communist hegemony and travel restrictions in Eastern Europe.

Eggert Briem was a graduate student at Aarhus at the time.

These meetings were the beginnings of decades-long collaboration and friendships encompassing all the families.

Lectures given in the year were published as the **Lecture Notes Series of the Matematiske Institut**; Bill's lectures were devoted to the Banach space $C(S)$, an important source.

More European visits

Bill returned to Aarhus for the year 1974–75 at the invitation of **Ebbe Thue Poulsen**.

The first mathematical visit of Bill and Elly to Britain was in January, 1975, when they came from Aarhus; this was a bracing experience in that cold winter, and they felt that some rooms in northern Britain were somewhat under-heated.

The final year-long visit to Denmark was for the year 1979–80, at the invitation of **Kjeld Laursen**.

After that Bill and Elly came quite few times to Leeds for collaborative work - and I spent several years enjoying their hospitality in Berkeley.

We collaborated by airmail letters, with a turnround time of about 3 weeks; no email then.

Conferences on Banach algebras

In 1974, Bill and Phil decided to organise a workshop at UCLA on **Homomorphisms and derivations from Banach algebras**. The idea of holding such a conference was fairly new.

The plan was to collect together for some time, and with some leisure, those interested in a particular topic, in this case, that of general Banach algebras and the structure and automatic continuity of homomorphisms and derivations from these algebras.

All participants were welcome to present their current work; open questions were discussed easily in a relaxed atmosphere; the hope, often fulfilled, was that progress would be made at the meeting and that contacts would be established that would flourish in the future.

Bill may have remembered the meeting at Oklahoma in 1950, and a meeting in Tulane in 1965, whose proceedings were influential.

The first meeting at UCLA

This took place from 1–12 July 1974. I was paid by NSF as a summer job to organise the conference.

The participants included **Richard Arens**, **Bill Bade**, **Andy Browder**, **Phil Curtis**, **Garth Dales**, **Fred Dashiell**, **Ted Gamelin**, **Sandy Grabiner**, **Kjeld Laursen**, **Peter McClure**, **Don Marshall**, **Robert Moore**, **Tony O'Farrell**, **Steve Scheinberg**, **Jim Stein**, **Marc Thomas**, and **Wiesław Żelazko**.

Conferences up to 1993

Conferences with organisers; these were the ones that Bill attended.

First conference - UCLA, 1–12 July 1974 (Bill Bade and Phil Curtis)

Second conference - Leeds, 14–18 June 1976 (Garth Dales)

Third conference - Long Beach 17–31 July 1981 (John Bachar, Bill Bade, Phil Curtis, Garth Dales)

Fourth conference - Copenhagen 15–26 July 1985 (Kjeld Laursen)

Fifth conference - Berkeley 21 July–1 August 1986 (Bill Bade and Phil Curtis)

Sixth conference - Leeds 23 June–3 July 1987 (Garth Dales)

Seventh conference - Berkeley 6–22 July 1988 (Bill Bade and Phil Curtis)

Eighth conference - Canberra 2–21 January 1989 (Rick Loy)

Ninth conference - Berkeley 23 July–3 August 1990 (Bill Bade and Phil Curtis)

Tenth conference - Cambridge 1–12 July 1991 (Graham Allan and Tom Ransford)

Eleventh conference - Winnipeg 3–13 August 1993 (Fereidoun Ghahramani and Peter McClure)

Thirteenth conference - Blaubeuren August 1997 (Ernst Albrecht and Martin Mathieu)

At the Winnipeg Conferences, Bill and Phil received presentations to mark their retirements and their roles as 'fathers of the Banach algebra conferences'.

Mathematical papers

I can pick out only a small selection.

Bill had various early papers on the operational calculus and spectral operators, based on the work of Taylor and Dunford.

For example, a bounded Boolean algebras $\mathfrak{B}$ of projections is said to be complete (now 'Bade-complete') if it contains the supremum and infimum of every family in $\mathfrak{B}$, where the suprema and infima exist in both a lattice-theoretical and topological sense, and it is shown that the weak and strong operator topologies coincide on a complete Boolean algebras of projections.

The Banach space $C(S)$

Bill gave various seminars on the Banach space $C(S)$ of all continuous functions on a compact space S, with the uniform norm. For example, he explained when $C(S)$ is isometrically a dual space.

There are notes from 1957; a (somewhat faded) copy is in the holdings of the Mathematics–Statistics library at Berkeley. They prove that, if $C(S)$ has an isometric predual, then S is hyper-Stonean in the sense of Dixmier. The converse direction is proved in his Aarhus notes.

These 1957 Notes were, for a long period, the only place where some basic facts about $C(S)$ were unified and expounded in Bill's very lucid and careful style.

Several graduate students made strong contributions to the theory. Some of the questions that they left open seem to be open to this day; a recent meeting in Warsaw heard of progress.

Homomorphisms from $C(K)$

At Yale in 1958–59, Bill and Phil attended a working group on an alleged positive solution to **Kaplansky's problem**: this was the conjecture by Kaplansky that all algebra norms on $C(K)$ for each compact space K are equivalent to the uniform norm, or, equivalently, that all homomorphisms from $C(K)$ into a Banach algebra are automatically continuous.

It was soon seen that the alleged proof was fallacious.

They made substantial progress, proving that such a homomorphism must be continuous when restricted to a certain dense subalgebra of $C(K)$.

Homomorphisms from $C(K)$ - more details

First they established a form of what is now known as the **Bade–Curtis main boundedness theorem**.

Theorem Let $\mathfrak{A}$ be a commutative Banach algebra, and let ν be a homomorphism from $\mathfrak{A}$ into a Banach algebra. Suppose that (g_n) and (h_n) are sequences of non-zero elements in $\mathfrak{A}$ such that $g_n h_n = g_n$ $(n \in \mathbb{N})$ and $h_m h_n = 0$ $(m, n \in \mathbb{N}, m \neq n)$. Then

$$\sup\{\|\nu(g_n)\| \,/\, \|g_n\| \, \|h_n\| : n \in \mathbb{N}\} < \infty \,.$$

Second, they proved that, in the case where $\mathfrak{A}$ is a regular Banach function algebra, the discontinuity of ν can be localized to a finite **singularity set**.

Homomorphisms from $C(K)$ - more details

These results have been substantially developed by Bill and Phil and many others in the context of Banach bimodules and a more general class of linear maps that includes homomorphisms, derivations, and intertwining maps, and are the foundation pillars of automatic continuity theory.

The theory of Bade and Curtis was extended by **Allan Sinclair** to show that, if there is a discontinuous homomorphism from a space of the form $C(K)$, then there is a non-maximal, prime ideal P in $C(K)$ with P contained in a unique maximal ideal M such that there is an embedding of the quotient algebra M/P into a radical Banach algebra.

There are also substantial generalizations of the theory to non-commutative C^*-algebras and group algebras by Sinclair, Laursen, and Runde.

Homomorphisms from $C(K)$ - an answer

During the year 1973–74 at UCLA, I attacked the problem of constructing such an embedding of M/P, eventually establishing that such an embedding exists provided that one is willing to assume the Continuum Hypothesis, CH. Bill studied my paper and made some very valuable comments (as the referee!).

The same result was proved independently at the same time by **Jean Esterle**.

It was proved by **Solovay** and **Woodin** that one cannot prove this theorem in ZFC: there are models of ZFC in which all homomorphisms from $C(K)$ are continuous.

Homomorphisms from $C(K)$ - further results

The subject still has life. For example, a strong result of **Hung Le Pham** shows that there are ideals which are not finite intersections of prime ideals, but which are kernels of discontinuous homomorphims.

Several questions about homomorphisms from general C^*-algebras are still open. For example, is every homomorphism with dense range from a C^*-algebra into a semisimple Banach algebra automatically continuous?.

Wedderburn decompositions

Here is the other paper written in Yale.

Let $\mathfrak{A}$ be an algebra with radical $\mathfrak{R}$. The paper was the first main attack on the question when the 'Wedderburn principal theorem' holds for a commutative Banach algebra $\mathfrak{A}$; an earlier example of Feldman had shown that this is not always true.

A Banach algebra $\mathfrak{A}$ is **decomposable** if there is a subalgebra $\mathfrak{B}$ with $\mathfrak{A} = \mathfrak{B} \oplus \mathfrak{R}$, and **strongly decomposable** if there is such a closed subalgebra $\mathfrak{B}$.

Bill and Phil proved that a commutative Banach algebra need not be decomposable, and that it may be decomposable without being strongly decomposable, and they obtained various 'uniqueness results'.

The separating space and continuity ideal

In 1973, Bill and Phil obtained conditions on a semisimple Banach algebra $\mathfrak{A}$ such that every derivation from $\mathfrak{A}$ into an arbitrary Banach $\mathfrak{A}$-bimodule is automatically continuous, building on a theorem of **John Ringrose**.

This paper made sustained use of the **separating space** and **continuity ideal** of an intertwining map, notions that are now basic in automatic continuity theory.

They left open the question whether all derivations from the disc algebra are automatically continuous; this was resolved negatively by myself - and this result led to the job at UCLA and many future visits to California.

Derivations from $C^{(n)}(\mathbb{I})$

In 1976, with Kjeld, Bill and Phil showed that any derivation from $C^{(n)}(\mathbb{I})$ into a Banach $C^{(n)}(\mathbb{I})$-bimodule must be continuous when restricted to $C^{(2n)}(\mathbb{I})$, where $C^{(2n)}(\mathbb{I})$ has its natural norm.

It was proved that a homomorphism from $C^{(n)}(\mathbb{I})$ into a Banach algebra that is continuous on $C^{(k)}(\mathbb{I})$ for some $k > n$ must be continuous on $C^{(2n+1)}(\mathbb{I})$; the index '$2n + 1$' cannot be reduced to $2n$.

They gave an attractive example of a natural derivation into a module $L^p(\mathbb{I})$ that is discontinuous on every dense subalgebra of $C^{(1)}(\mathbb{I})$.

The prime ideal theorem

Bill and Phil proved in 1978 the important **prime ideal theorem**, showing that, when there is a discontinuous intertwining map from a Banach algebra, there is also one whose left continuity ideal is a prime ideal.

This leads to a strong theorem giving conditions for all intertwining maps from a commutative Banach algebra which has no closed, prime ideal of infinite codimension to be continuous.

Divisible subspaces

Let A be a Banach algebra, and take $a \in A$. A linear subspace D of A is a-**divisible** if

$$(a - z)D = D$$

for each $z \in \mathbb{C}$; such divisible subspaces are a key feature of **Graham Allan**'s embeddings of the algebra of all formal power series in one variable into certain Banach algebras.

In 1980, Bill, Phil, and Kjeld obtained a sequence of impressive results that show that, in certain circumstances, all homomorphisms between two specific Banach algebras are automatically continuous whenever the range algebra has no non-trivial divisible subspaces.

Convolution algebras

Let ω be weight function on $\mathbb{R}^+ = [0, \infty)$, and let $L^1(\omega)$ consist of the measurable functions integrable against the weight $\omega(t)\mathrm{d}t$, so that $L^1(\mathbb{R}^+, \omega)$ is a Banach algebra for the convolution product, $\star$. Obvious 'standard (closed) ideals' are those consisting of functions that vanish on $[0, a]$ for $a > 0$.

The question is: Are there any more when ω is 'radical'.

In 1981, Bill and I found rather a lot of relevant examples.

Soon afterwards, **Yngve Domar** showed that all closed ideals in most radical algebras $L^1(\mathbb{R}^+, \omega)$ are indeed standard.

Weak amenability

Let A be a Banach algebra. Then we all know, after Barry Johnson, that A is **amenable** if every continuous derivation into a dual module is inner.

In 1987, Bill and Phil and I introduced the notion that A is **weakly amenable**: the only module that you look at is the dual of A itself.

We determined, for example, that the Beurling algebras $\ell^1(\mathbb{Z}, (1+|n|)^\alpha)$ are weakly amenable if and only if $0 \leq \alpha < 1/2$.

Derivations from integral domains

In 1987, in Leeds, Bill and I returned to the algebras $L^1(\mathbb{R}^+, \omega)$. Earlier results left open the question of the automatic continuity of all derivations from commutative Banach algebras which do have a closed, prime ideal of infinite codimension; the algebras $L^1(\mathbb{R}^+, \omega)$ satisfy this condition, and hence they seem to be a good test case for the general question.

We proved that all derivations from such algebras have strong continuity properties, but it remains open whether or not they must indeed all be continuous.

A challenge for the young!

Decompositions

During Bill's visit to Leeds in 1993, we were joined by **Zinaida Lykova** from Moscow.

Let A be a Banach algebra, and let

$$\Sigma : 0 \to I \to \mathfrak{A} \to A$$

be an extension of A. When does each such extension split or split strongly?

This led to Bill's longest research memoir, in the MAMS.

The memoir develops the theory in the setting of the second-order continuous cohomology group $\mathcal{H}^2(A, E)$, where E is a Banach A-bimodule, and gives results when A is a (commutative) C^*-algebra, the disc algebra, or the algebra $C^{(n)}(\mathbb{I})$, for example.

A beautiful finale

Bill's final paper, with Phil Curtis and Allan Sinclair, is a delightful paper of 2000.

Let $\Sigma : 0 \to \mathfrak{R} \to \mathfrak{A} \to C(K)$ be an extension of a space $C(K)$, where $\mathfrak{A}$ is a commutative Banach algebra and $\mathfrak{R}$ is the radical of $\mathfrak{A}$.

Forty years earlier, Bill and Phil had proved that Σ splits strongly when $\mathfrak{R}$ is nilpotent and K is totally disconnected, and they wondered if the result held for each compact space K.

This conjecture was finally proved by **Ernst Albrecht** and **Olaf Ermert** in 1997 by using the theory of Arens products on the second duals of Banach algebras.

Bill, Phil, and Allan gave a direct proof under weaker hypotheses.

This was a pleasing finale to the break-through year of 1958–59.