FOURIER ANALYSIS PREP!

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To make sure that you're prepared for this course, we've put together a few questions. The purpose is to help you check whether you are up to speed with the pre-requisites you'll need to stay abreast in this course. If you have any questions about what to review, please let me (Julie) know! Happy to help so that you'll succeed in the course.

1. Linear Algebra

(1) Give an example of an orthonormal basis for $\mathbb{R}^3$ which is *not* the standard one.

(2) Let's call your basis vectors now u, v, and w. Write the vector

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = au + bv + cw,$$

for $a, b, c \in \mathbb{R}$.

(3) Let $x, y \in \mathbb{R}^n$ for some $n \geq 2$ be non-zero vectors. What can you say about x and y, geometrically, if you know that

$$x \cdot y = 0?$$

(4) What geometric information does

$$\sqrt{x \cdot x}$$

tell you about the vector, x?

2. Analysis

(1) Determine whether or not these converge, and in case of convergence, compute the limit

$$\lim_{x \to 0} x \tan\left(\frac{1}{x}\right)$$

$$\lim_{x \to \frac{\pi}{2}} \frac{e^x - \sin(x)}{x - \frac{\pi}{2}}$$

$$\lim_{x \to 0} \frac{\tan(5x)}{x}$$

$$\lim_{x \to 0} x \sin\left(\frac{1}{x}\right)$$

$$\lim_{x \to \infty} \frac{6x^8 + e^x}{3^{2x} + 500x}$$

$$\sum_{n=0}^{\infty} \frac{5^n}{n!}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$$

(2) Compute:

$$\partial_x(\sin(e^{x-y})), \quad \partial_y(\sin(e^{x-y})), \quad \partial_t(\sin(e^{x-y})).$$

(3) Assume that f depends on a single variable called t which is *different and independent from* a variable called z. Assume that g depends on this variable called z (and *only* on this variable, z). If

$$f(x) = g(z),$$

then what can you conclude about f and g?

(4) Compute:

$$\int_0^2 xe^{inx}dx, \quad \text{for } n \in \mathbb{Z}.$$

(5) Compute:

$$\int_0^L \sin^2(x)dx.$$

(6) What does it really mean to write

$$\int_{\mathbb{R}} f(x)dx$$

or

$$\int_0^{\infty} f(x)dx?$$

(7) Compute:

$$\int_0^{\infty} \frac{x}{e^x}dx.$$

(8) Compute:

$$\int_{\mathbb{R}} \frac{\sin(x)}{x}dx.$$

If you're not sure about any of these problems, don't worry! There's still time (but it is ticking away...). So, if you need any help, please just ask!