MATEMATIK
Chalmers Tekniska Hogskola

Dugga

Fourieranalys/Fourier Metoder, lp1, 2017

Skriv ditt namn och personnummer - *tydligt!*

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1. (1P) Define the Fourier series of a function on $[-\pi, \pi]$. This includes defining the Fourier coefficients. For a function $f(x)$ defined on $[-\pi, \pi]$ the Fourier coefficients:
$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) e^{-inx} dx,$$
and the Fourier series is
$$\sum_{n \in \mathbb{Z}} c_n e^{inx}.$$

2. (1P)The Fourier series for $f(x) = x$ for the function $x \in (-\pi, \pi)$ is
$$\sum_{n \geq 1} \frac{(-1)^{n+1}}{n} \sin(nx).$$
Evaluate
$$\sum_{n \geq 1} \frac{(-1)^{n+1}}{n} \sin(3\pi n/2).$$
We see here that $x = \frac{3\pi}{2}$. This is outside of the interval $(-\pi, \pi)$. So we need to remember the whole copy-paste business: the Fourier series corresponds to the function inside the interval, but outside it is 2π periodic. To figure out the value of the series we observe that $3\pi/2 - 2\pi = -\pi/2$. This is inside the interval $(-\pi, \pi)$. The function f is continuous in the interval $(-\pi, \pi)$. Hence the series converges to the value of the function at $-\pi/2$. Therefore:
$$\sum_{n \geq 1} \frac{(-1)^{n+1}}{n} \sin(3\pi n/2) = -\frac{\pi}{2}.$$

3. (1P) Define the Fourier transform. For $f(x)$ in $\mathcal{L}^2$ or $\mathcal{L}^1$ we can define
$$\hat{f}(\xi) = \int_{\mathbb{R}} f(x) e^{-ix\xi} dx.$$

4. (1P) Define convolution. For f and g both in $\mathcal{L}^2$ we can define:
$$f * g(x) = \int_{\mathbb{R}} f(x-y) g(y) dy.$$

5. (1P) What do you use to solve a PDE on $\mathbb{R}$: a Fourier series or the Fourier transform? What do you use to solve a PDE on $[-42, 42]$: a Fourier series or the Fourier transform?
On $\mathbb{R}$ we use the Fourier transform. On $[-42, 42]$ we use a Fourier series.