MATEMATIK
Chalmers Tekniska Hogskola

Dugga

Fourieranalys/Fourier Metoder, lp3, 2019

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1. (1P) To solve the initial value problem for the heat equation on $\mathbb{R}$ one can use
   (a) the Fourier transform
   (b) Fourier series
   (c) neither of these
   (d) both (a) and (b) will work.

   The correct answer is A.

2. (1P) To solve the initial value problem for the wave equation on $[0, 1]$ one can use
   (a) the Fourier transform
   (b) Fourier series
   (c) neither of these
   (d) both (a) and (b) will work.

   The correct answer is B.

3. (1P) Assume f is continuously differentiable (C^1) on $[-\pi, \pi]$. Let
   $$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{-inx} f(x)dx.$$
   What can be said about
   $$\sum_{n\in\mathbb{Z}} c_n?$$
   (a) the series converges to
   $$\frac{f(\pi) + f(-\pi)}{2}$$
   (b) there is insufficient information about f to determine whether or not the series converges
   (c) the series converges because it is bounded above by
   $$\int_{-\pi}^{\pi} |f(x)|^2 dx$$
   (d) the series converges to $f(0)$

   We can see that this is the Fourier series for f evaluated at the point 0. That's because
   $$\sum_{n\in\mathbb{Z}} c_n e^{i0n} = \sum_{n\in\mathbb{Z}} c_n.$$
   By the theorem on the pointwise convergence of Fourier series, and since f is continuously differentiable and thus continuous at 0, the Fourier series converges to $f(0)$. So D is the correct answer.

4. (1P) Assume that f is continuous on $[-\pi, \pi]$. Define c_n as in the previous problem. What can be said about
   $$\sum_{n\in\mathbb{Z}} |c_n|^2?$$

(a) the series converges and is equal to

$$\int_{-\pi}^{\pi} |f(x)|^2 dx$$

(b) the series converges and is equal to

$$\frac{f(\pi) + f(-\pi)}{2}$$

(c) the series converges because it is bounded above by:

$$\int_{-\pi}^{\pi} |f(x)|^2 dx$$

(d) none of these are correct

By the assumption that f is continuous on the closed interval, it is also bounded there. Consequently it is in $\mathcal{L}^2(-\pi, \pi)$. Bessel's inequality therefore tells us that

$$\sum_{n\in\mathbb{Z}} |c_n|^2 \le \frac{1}{2\pi} \int_{-\pi}^{\pi} |f(x)|^2 dx \le \int_{-\pi}^{\pi} |f(x)|^2 dx,$$

since $\frac{1}{2\pi} < 1$. So the correct answer is C.

5. (1P) Assume that f is in a Hilbert space, H. Let $\{\phi_n\}_{n\in\mathbb{N}}$ be an orthonormal set in H. Let $\hat{f}_n = \langle f, \phi_n \rangle$. What can be said about

$$\sum_{n\in\mathbb{N}} \hat{f}_n?$$

(a) it is equal to f

(b) it is the "best approximation" of f

(c) both (a) and (b) are correct

(d) neither (a) nor (b) is correct

The thing above, $\sum_{n\in\mathbb{N}} \hat{f}_n$, is a number. It is not an element of H. It is just a number. So D is the correct answer here.