MATEMATIK
Chalmers Tekniska Hogskola

Dugga

Fourieranalys/Fourier Metoder, lp1, 2019

Skriv ditt namn och personnummer - *tydligt!*

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1. (1P) To solve the initial value problem for the heat equation on $\mathbb{R}$ one can use
   (a) the Fourier transform
   (b) Fourier series
   (c) neither of these
   (d) both (a) and (b) will work.

2. (1P) To solve the initial value problem for the wave equation on $[0, 1]$ one can use
   (a) the Fourier transform
   (b) Fourier series
   (c) neither of these
   (d) both (a) and (b) will work.

3. (1P) Assume f is continuously differentiable (C^1) on $[-\pi, \pi]$. Let

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{-inx} f(x) dx.$$

   What can be said about

$$\sum_{n \in \mathbb{Z}} c_n?$$

   (a) the series converges to

$$\frac{f(\pi) + f(-\pi)}{2}$$

   (b) there is insufficient information about f to determine whether or not the series converges
   (c) the series converges because it is bounded above by

$$\int_{-\pi}^{\pi} |f(x)|^2 dx$$

   (d) the series converges to $f(0)$

4. (1P) Assume that f is continuous on $[-\pi, \pi]$. Define c_n as in the previous problem. What can be said about

$$\sum_{n \in \mathbb{Z}} |c_n|^2?$$

(a) the series converges and is equal to

$$\int_{-\pi}^{\pi} |f(x)|^2 dx$$

(b) the series converges and is equal to

$$\frac{f(\pi) + f(-\pi)}{2}$$

(c) the series converges because it is bounded above by:

$$\int_{-\pi}^{\pi} |f(x)|^2 dx$$

(d) none of these are correct

5. (1P) Assume that f is in a Hilbert space, H. Let $\{\phi_n\}_{n \in \mathbb{N}}$ be an orthonormal set in H. Let $\hat{f}_n = \langle f, \phi_n \rangle$. What can be said about

$$\sum_{n \in \mathbb{N}} \hat{f}_n?$$

(a) it is equal to f

(b) it is the "best approximation" of f

(c) both (a) and (b) are correct

(d) neither (a) nor (b) is correct