ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS FOR CHEMISTS

JULIE ROWLETT

1. Classification of eODEs

Exercise 1. Determine the degree of these eODEs, and also whether or not they are linear:

$$y' = 1 + y^2$$
$$y' = ay(b - y)$$
$$tx\dot{x} = 1$$
$$y' = xy$$
$$y' = 1 - y^2$$
$$x^2y' + y = 0$$
$$y''' + 3y'' + 3y' + y = 0$$
$$y'''' + 4y''' + 6y'' + 4y' + y = 0$$

Exercise 2. Determine in each case the eODE operator, L, and its order. Is L linear or not? Is the eODE homogeneous or not?

(1) $u' + u'' = 0.$
(2) $e^u + 1 = 0$
(3) $4x^2u''(x) + 12xu'(x) + 3u(x) = 0.$
(4) $2tu'4u = 3$
(5) $\frac{u'(x)}{u(x)} = e^x$
(6) $u'(x) = \frac{x}{u(x)}$
(7) $u''(x) = 5$
(8) $u'(x) = x^2$
(9) $u'(x) + 5u(x) = 2$
(10) $u'' = -u$

Exercise 3. Classify the heat equation, wave equation, and Laplace equation.

Exercise 4. Classify the following equations:

(1) $u_t = u_{xx} + 2u_x + u$
(2) $u_t = u_{xx} + e^{-t}$
(3) $u_{xx} + 3u_{xy} + u_{yy} = \sin(x)$
(4) $u_{tt} = uu_{xxxx} + e^{-t}$

Exercise 5. Investigate solutions of the form

$$u(x, t) = e^{ax+bt}$$

to the equation

$$u_t = u_{xx}.$$

Exercise 6. Solve:

$$\frac{\partial u(x,y)}{\partial x} = 0.$$

Exercise 7. Solve:

$$\frac{\partial^2 u(x,y)}{\partial x \partial y} = 0.$$

Compare with the eODE $u''(t) = 0$. How many solutions are there to the ODE, and what are they? How many solutions are there to the PDE (above)? Describe them.

Exercise 8. Compare the preceding two exercises with the eODE:

$$y''(x) = 0,$$

otherwise written as

$$\frac{d^y}{dx^2} = 0.$$