Lösningar Flervariabelmatematik 2 27/5-14

1) $\nabla f = \frac{1}{x+x^2y}\left(1+2xy,\ x^2\right)$

$\nabla f(1,2) = \frac{1}{3}\left(5,1\right)$

Störst tillväxt $\frac{1}{3}\sqrt{5^2+1^2} = \underline{\underline{\frac{\sqrt{26}}{3}}}$ i riktning $\underline{\underline{(5,1)}}$

Tillväxt i riktning $(1,2)$ $\quad \frac{1}{3}\cdot\frac{1}{\sqrt{5}}(5,1)\bullet(1,2) = \underline{\underline{\frac{7}{3\sqrt{5}}}}$

tangentplanet:

$$z - \ln 3 = \frac{5}{3}(x-1) + \frac{1}{3}(y-2)$$

$$\underline{\underline{z = \frac{5}{3}x + \frac{1}{3}y + \ln 3 - \frac{7}{3}}}$$

2) $\begin{pmatrix} x \\ x' \end{pmatrix}' = \begin{pmatrix} x' \\ t^2 - x' - e^x \end{pmatrix}$

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f = @(t,x) [x(2), t^2 - x(2) - exp(x(1))]'
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3) $y = A\cos(bx+\varphi)$

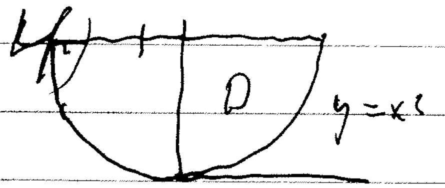
$$\begin{cases} A\cos(\cancel{bx+}\varphi) - 1 = 0 \\ A\cos(b+\varphi) - 2 = 0 \\ A\cos(2b+\varphi) - 1 = 0 \end{cases}$$

kurvan genom 3 punkter

Newton's metod

$$\begin{pmatrix} A_{n+1} \\ b_{n+1} \\ \varphi_{n+1} \end{pmatrix} = \begin{pmatrix} A_n \\ b_n \\ \varphi_n \end{pmatrix} - \begin{pmatrix} \cos(\varphi_n), & 0, & -A\sin\varphi_n \\ \cos(b_n+\varphi_n), & -A\sin(b_n+\varphi_n), & -A\sin(b_n+\varphi_n) \\ \cos(2b_n+\varphi_n), & -2A\sin(2b_n+\varphi_n), & -A\sin(2b_n+\varphi_n) \end{pmatrix}^{-1}$$

$$\begin{pmatrix} A_n\cos\varphi_n - 1 \\ A_n\cos(b_n+\varphi_n) - 2 \\ A_n\cos(2b_n+\varphi_n) - 1 \end{pmatrix}$$

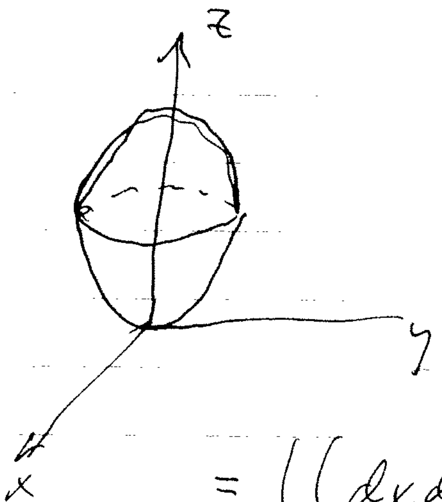


$$\iint_D x^2 y \, dx\, dy =$$

$$= \int_{-1}^{1} dx \int_{x^2}^{1} x^2 y \, dy = \int_{-1}^{1} x^2 \frac{y^2}{2}\Big|_{x^2}^{1} dx =$$

$$= \frac{1}{2}\int_{-1}^{1} x^2 - x^6 \, dx = \cancel{1}\left(\frac{1}{3} - \frac{1}{7}\right) = \frac{4}{\cancel{1}\cdot 21} = \frac{4}{21}$$

5)



$x^2+y^2 = 2-x^2-y^2, \quad 2(x^2+y^2)=2$

$$\iiint_D z\,dx\,dy\,dz =$$

$$= \iint_{x^2+y^2\le 1} dx\,dy \int_{x^2+y^2}^{2-x^2-y^2} z\,dz = \iint_{x^2+y^2\le 1} \frac{z^2}{2}\Big|_{x^2+y^2}^{2-x^2-y^2} dx\,dy =$$

$$= \int_0^{2\pi} d\theta \int_0^1 \frac{r}{2}\left((2-r^2)^2 - r^4\right) dr =$$

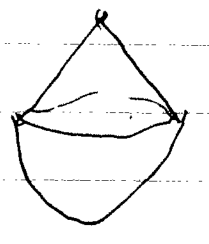
$$= \frac{2\pi}{2}\int_0^1 4r-4r^3\,dr = 4\pi\left(\frac{1}{2}-\frac{1}{4}\right) = \underline{\underline{\pi}}$$

6) Skjæring: $x^2+y^2 = 2-\sqrt{x^2+y^2}$

$$r^2 = 2-r$$

$$r^2+r-2=0$$

$$r = -\frac{1}{2} \pm \sqrt{\frac{1}{4}+2} = -\frac{1}{2} \pm \frac{3}{2} = \begin{cases} 1 \\ -2 \end{cases}$$



Volumen =

$$\iint_{x^2+y^2\le 1} 2-\sqrt{x^2+y^2}-(x^2+y^2)\,dx\,dy =$$

$$= \int_0^{2\pi} d\theta \int_0^1 r(2-r-r^2)\,dr = 2\pi\int_0^1 2r-r^2-r^3\,dr =$$

$$= 2\pi\left(r^2-\frac{r^3}{3}-\frac{r^4}{4}\right)\Big|_0^1 = 2\pi\left(1-\frac{1}{3}-\frac{1}{4}\right) = \underline{\underline{\frac{5\pi}{6}}}$$

7) Sökt: φ s.a.

$$\varphi'_x = e^{xy}(xy + xy^2 + y + 1) \qquad (1)$$

$$\varphi'_y = e^{xy}(x^2 + x^2y + x) \qquad (2)$$

(2) ser $\varphi'_y = e^{xy}x^2 + e^{xy}x^2y + e^{xy}x$

$$\varphi = e^{xy}x + e^{xy}xy - \int e^{xy}x\,dx + \int e^{xy}x\,dx$$

$$= e^{xy}x + e^{xy}xy + g(x)$$

$$\varphi'_x = e^{xy} + e^{xy}xy + e^{xy}y + e^{xy}xy^2 + g'(x)$$

jämförelse med (1) ser $g'(x) = 0$

$g(x) = 0$ t.ex.

$$\varphi = e^{xy}(x + xy)$$

$$\int_{(1,2)}^{(2,3)} P\,dx + Q\,dy = \varphi(2,3) - \varphi(1,2) = e^6(2+6) - e^2(1+2) =$$

$$= \underline{\underline{8e^6 - 3e^2}}$$

γ) $u = x^2 + y^2, \quad v = xy$

$$f'_x = f'_u 2x + f'_v y$$

$$f'_y = f'_u 2y + f'_v x$$

$$x f'_x - y f'_y = f'_u 2x^2 + f'_v xy - f'_u 2y^2 - f'_v xy$$

$$= (2x^2 - 2y^2) f'_u = 2x^2 - 2y^2$$

$$f'_u = 1$$

$$f = u + g(v)$$

$$\underline{\underline{f(x,y) = x^2 + y^2 + g(xy)}}$$

$$f(x,x) = x^2 + x^2 + g(x^2) = 0$$

$$g(x^2) = -2(x^2)$$

$$\underline{\underline{f(x,y) = x^2 + y^2 - 2xy = (x-y)^2}}$$