Tentamen i MVE041 Flervariabelmatematik

Den 28 May 2016, kl. 830-1230

Hjälpmedel: Formelblad (bifogat), inga räknare.

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Solutions

Tentamen består av en gödkantdel (38 poäng) och överbetygsdel (12 poäng), sammanlagt 50 poäng. För godkänt på tentamen krävs 25 poäng på gödkantdel. För betyg 4 krävs totalt 33 poäng var av minst 4 på överbetygsdel. För betyg 5 krävs totalt 43 poäng var av minst 7 på överbetygsdel. Bonuspoäng från 2016 räknas in i totala poängen. För att bli godkänd på hela kursen måsta du klara tentamen och alla 6 Matlab laborationer.

Lösningar läggs ut på kursens webbsida första vardagen efter tentamensdagen. Tentan rättas och bedöms anonymt. Resultat meddelas via Ladok ca. tre veckor efter tentamenstillfället. Första granskningstillfälle meddelas på kurswebbsidan, efter detta sker granskning alla vardagar 9-13, MV:s exp.

Godkäntdel

1. Betrakta funktionen $f(x,y) = \frac{1}{4}x^2 - y^2$.
   (a) (3 p) Bestäm gradienten $\bar{\nabla} f(x,y)$, och beräkna den i punkten $(4,3)$.
   (b) (2 p) Skriv en parametrering för $z = -5$ nivåkurvan av $f(x,y)$ i halv-planet $y \geq 0$.
   (c) (1 p) Skissa nivåkurvan från (a), dessutom gradientvektorn i den givna punkten.

2. (6 p) I vilken punkt (x,y) är funktionen $f(x,y) = -x^2 - y^2 + xy + 3x + y$ med den bivillkoret $x + y = 2$ störst?

3. (5 p) Bestäm integralen av $15y$ över området i första kvadranten som begränsas av $x = 0, y = x^2$, och $y = -x + 2$.

4. Betrakta den andra ordningens differentialekvationen $\frac{d^2u}{dt^2} = u^2 - 1$.
   (a) (1 p) Skriv den som ett första ordningens system av differentialekvationer med hjälp av $v = \frac{du}{dt}$.
   (b) (2 p) Betrakta en integreralkurva av fasvektorn som börjar vid $t = 0$ från punkten $(u,v) = (2,0)$. Vad är tangentvektorn till denna integralkurvan vid $t = 0$?
   (c) (2 p) Hitta fixpunkterna i fasvektorfältet.

5. (5 p) Integrera $\bar{\mathbf{F}}(x,y) = (\frac{2}{3}x)\hat{\imath} + (1 + \frac{3}{2}y)\hat{\jmath}$ längs kurvan $x^2 + \frac{9}{4}y^2 = 9$ från punkten $(3,0)$ till $(0,2)$.

6. (5 p) Betrakta $\bar{\mathbf{F}}(x,y) = (e^{x^2+y})\hat{\imath} + (e^{x^2+y} + y^3)\hat{\jmath}$ definerad på planet $\mathbb{R}^2$. Bestäm div$\bar{\mathbf{F}} = \nabla \cdot \bar{\mathbf{F}}$ och curl$\bar{\mathbf{F}} = \nabla \times \bar{\mathbf{F}}$. Är $\bar{\mathbf{F}}(x,y)$ *virvelfritt, källfritt* eller *konservativt*?

7. (6 p) Bestäm flödet av $\bar{\mathbf{F}}(x,y,z) = x\hat{\imath} + y\hat{\jmath} + (z+1)\hat{\mathbf{k}}$ ut ur den del av sfären $x^2 + y^2 + z^2 = 4$ som ligger i den första oktanten $\{x \geq 0, y \geq 0, z \geq 0\}$.

Överbetygsdel

8. (4 p) Bevisa Greens sats i planet på triangeln med hörn $(0,0), (1,0), (1,1)$.

9. (4 p) Avgör om följande påståenden är sanna eller falska, samt motivera ditta svar. (Inga poäng för rätt svar med felaktig/ingen motivering). Om $\bar{\mathbf{F}}(x,y,z) = F^1(x,y,z)\hat{\imath} + F^2(x,y,z)\hat{\jmath} + F^3(x,y,z)\hat{\mathbf{k}}$ är ett glatt konservativt vektorfält i $\mathbb{R}^3$, då är:
   (a) curl$\bar{\mathbf{F}} = \bar{\nabla} \times \bar{\mathbf{F}}$ icke-försvinnande.
   (b) $\oint_C \bar{\mathbf{F}} \cdot d\bar{\mathbf{r}} = 0$ för godtycklig sluten kurva C.

10. (4 p) Använda divergens-satsen för att bestämma flödet av vektorfältet $\bar{\mathbf{F}} = \frac{x}{18}\sqrt{9x^2 + 4y^2}\hat{\imath} + \frac{y}{18}\sqrt{9x^2 + 4y^2}\hat{\jmath} - 1\hat{\mathbf{k}}$ ut ur den slutna ytan bildad av den elliptiska paraboloiden $z = 6 - \frac{3}{2}x^2 - \frac{2}{3}y^2$ och planet $z = 0$.

Exam for MVE041 Flervariabelmatematik

28 May 2016, kl. 830-1230

Help materials: Attached formula sheet. No calculators.

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Solutions

The exam consists of a passing section (38 points) and mastery section (12 points), for a total of 50 points. A passing grade on the exam is obtained with a score of 25 points from the passing part. A grade 4 is obtained with 33 points in total, and at least 4 from the mastery section. A grade 5 is obtained with 43 points in total, and at least 7 from the mastery section. Bonus points earned during the 2016 course are included in the total scores. To pass the entire course you must pass the exam and complete all six laborations.

Solutions will be posted on the course website on the first weekday following the exam. The exam is graded anonymously. Results are available on Ladok starting three weeks after the exam day. The first day on which you may contest your grade will be posted on the course website, and after that you may file a contest with the MV exp weekdays 9-13.

Passing Part

1. Consider the function $f(x,y) = \frac{1}{4}x^2 - y^2$.
   (a) (3 p) Compute the gradient $\bar{\nabla} f(x,y)$, and evaluate your expression at the point $(4,3)$.
   (b) (2 p) Write a parametrization for the $z = -5$ level curve of $f(x,y)$ in the $y \geq 0$ half-plane.
   (c) (1 p) Sketch the level curve from part (a), and also the gradient vector at the given point.

2. (6 p) At which point (x,y) is the function $f(x,y) = -x^2 - y^2 + xy + 3x + y$ subject to the constraint $x + y = 2$ maximized?

3. (5 p) Compute the integral of $15y$ over the region in the first quadrant bounded by $x = 0, y = x^2$, and $y = -x + 2$.

4. Consider the second order differential equation $\frac{d^2u}{dt^2} = u^2 - 1$.
   (a) (1 p) Write this as a first order system of differential equations using $v = \frac{du}{dt}$.
   (b) (2 p) Consider an integral curve of the phase vector field which starts at $t = 0$ from the point $(u,v) = (2,0)$. What is the tangent vector to this integral curve at $t = 0$?
   (c) (2 p) Find any fixed points in the phase-vector field.

5. (5 p) Integrate $\bar{\mathbf{F}}(x,y) = (\frac{2}{3}x)\hat{\imath} + (1 + \frac{3}{2}y)\hat{\jmath}$ over the curve $x^2 + \frac{9}{4}y^2 = 9$ from the point $(3,0)$ to $(0,2)$.

6. (5 p) Consider $\bar{\mathbf{F}}(x,y) = (e^{x^2+y})\hat{\imath} + (e^{x^2+y} + y^3)\hat{\jmath}$ defined on the plane $\mathbb{R}^2$. Compute $\text{div}\bar{\mathbf{F}} = \nabla \cdot \bar{\mathbf{F}}$ and $\text{curl}\bar{\mathbf{F}} = \nabla \times \bar{\mathbf{F}}$. Is $\bar{\mathbf{F}}(x,y)$ *irrotational, solenoidal* or *conservative*?

7. (6 p) Find the flux of $\bar{\mathbf{F}}(x,y,z) = x\hat{\imath} + y\hat{\jmath} + (z+1)\hat{\mathbf{k}}$ out of the portion of the sphere $x^2 + y^2 + z^2 = 4$ which lies in the first octant $\{x \geq 0, y \geq 0, z \geq 0\}$.

Mastery Part

8. (4 p) State and prove Green's theorem in the plane on the triangle with corners $(0,0), (1,0), (1,1)$.

9. (4 p) Decide whether the following statements are true or false, and motivate your answer. (No points for correct answer but incorrect/no argument) If $\bar{\mathbf{F}}(x,y,z) = F^1(x,y,z)\hat{\imath} + F^2(x,y,z)\hat{\jmath} + F^3(x,y,z)\hat{\mathbf{k}}$ is a smooth conservative vector field in $\mathbb{R}^3$, then:
   (a) $\text{curl}\bar{\mathbf{F}} = \bar{\nabla} \times \bar{\mathbf{F}}$ is non-vanishing.
   (b) $\oint_C \bar{\mathbf{F}} \cdot d\bar{\mathbf{r}} = 0$ for any closed curve C.

10. (4 p) Using the divergence theorem find the flux of the vector field $\bar{\mathbf{F}} = \frac{x}{18}\sqrt{9x^2 + 4y^2}\hat{\imath} + \frac{y}{18}\sqrt{9x^2 + 4y^2}\hat{\jmath} - 1\hat{\mathbf{k}}$ out of the closed surface formed from the elliptic parabola $z = 6 - \frac{3}{2}x^2 - \frac{2}{3}y^2$ and the plane $z = 0$.

#1

$$f(x,y) = \frac{1}{4}x^2 - y^2$$

a) $\bar{\nabla} f = \frac{1}{2} x \,\hat{\imath} - 2y\,\hat{\jmath}$

$\bar{\nabla} f(4,3) = 2\hat{\imath} - 6\hat{\jmath}$

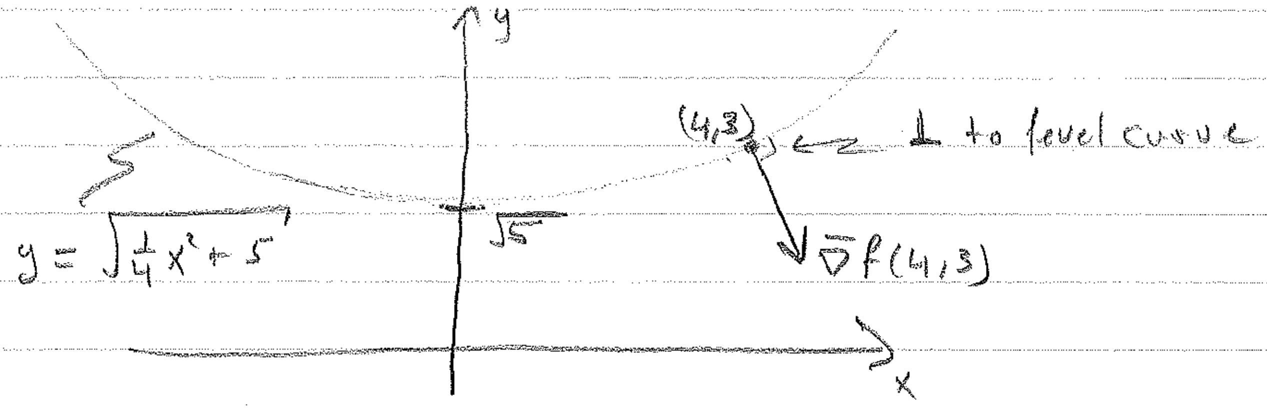
b) The $z = -5$ level curve of the graph of $f(x,y)$ is $-5 = \frac{1}{4}x^2 - y^2$

$\Rightarrow \quad y^2 = \frac{1}{4}x^2 + 5 \qquad$ Hyperbola.

In $y \geq 0$ plane $\quad y = +\sqrt{\frac{1}{4}x^2 + 5}$

$$\boxed{\Rightarrow \bar{r}(t) = t\,\hat{\imath} + \sqrt{\frac{1}{4}t^2 + 5}\;\hat{\jmath}, \quad -\infty < t < \infty}$$

c)



Note Another parameterization is

$\bar{r}(t) = \sqrt{20}\,\sinh(t)\,\hat{\imath} + \sqrt{5}\,\cosh(t)\,\hat{\jmath}, \quad -\infty < t < \infty$

Others possible

#2

$$L(x,y,\lambda) = -x^2 - y^2 + xy + 3x + y + \lambda(x+y-2)$$

$$0 = \frac{\partial L}{\partial x} = -2x + y + 3 + \lambda \qquad (A)$$

$$0 = \frac{\partial L}{\partial y} = -2y + x + 1 + \lambda \qquad (B)$$

$$0 = \frac{\partial L}{\partial \lambda} = x + y - 2 \qquad (C)$$

$$2x - y - 3 \underset{(A)}{=} \lambda \underset{(B)}{=} 2y - x - 1$$

$$\Rightarrow 3x - 3y - 2 = 0$$

$$+ \quad 3x + 3y = 6 \quad \text{from } (C)$$

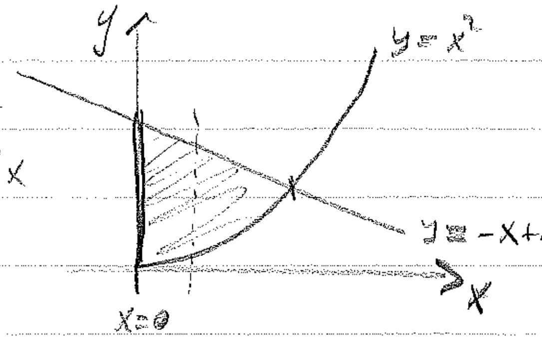
$$6x = 8 \quad \Rightarrow \boxed{x = 4/3}$$

$$(C) \Rightarrow y = 2 - 4/3 = 2/3 \qquad \boxed{y = 2/3}$$

$$\boxed{\text{Thus, } f(x,y) \text{ is maximized at } (x,y) = (4/3, 2/3)}$$

#3

$$\int_{x=0}^{1}\int_{y=x^2}^{-x+2} 15y\,dy\,dx = \int_{x=0}^{1}\left(\frac{15}{2}y^2\right)\Bigg|_{x^2}^{-x+2} dx$$



$$= \frac{15}{2}\int_0^1\left((-x+2)^2 - x^4\right)dx$$

$x^2 = -x+2 \Rightarrow x^2+x-2=0$

$(x-1)(x+2)=0$

$$= \frac{15}{2}\int_0^1\left(x^2-4x+4-x^4\right)dx$$

$$= \frac{15}{2}\left(\frac{1}{3}-2+4-\frac{1}{5}\right)$$

$$= \frac{5}{2}+15-\frac{3}{2} \qquad \boxed{=16}$$

#4

$\frac{d^2u}{dt^2} = u^2-1$, if $v = \frac{du}{dt}$

$$\begin{cases}\frac{du}{dt} = v \\ \frac{dv}{dt} = u^2-1\end{cases} \Rightarrow \bar{F} = v\hat{\imath} + (u^2-1)\hat{\jmath}$$

phase vector field.

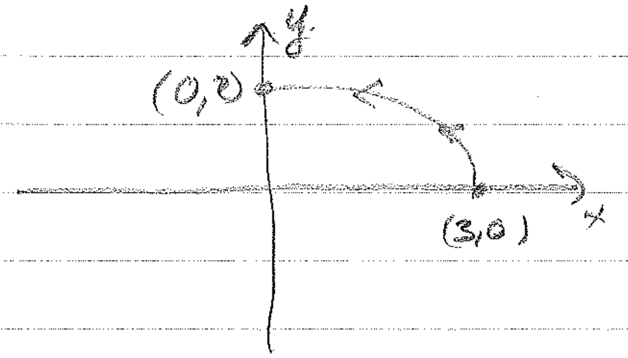
Integral curve satisfies $\frac{d\bar{r}}{dt} = \bar{F}$

at $(u,v) = (2,0)$ $\boxed{\bar{F} = 0\hat{\imath} + 3\hat{\jmath}}$

$\boxed{\text{Fixed points: } F(u,v) = 0 \text{ at } v=0,\ u=\pm 1}$

#5

$$\int_C \bar{F}\cdot d\bar{r} \quad , \quad \bar{F} = \frac{2}{3}x\,\hat{i} + \left(1+\frac{3}{2}y\right)\hat{j}$$



$C: \ x^2 + \frac{9}{4}y^2 = 9$

$\Rightarrow \ x^2/9 + y^2/4 = 1$

$\Rightarrow \ x = 3\cos t, \ y = 2\sin t$

So the curve is parameterized by $\bar{r}(t) = 3\cos t\,\hat{i} + 2\sin t\,\hat{j}$

$0 \le t \le \pi/2$

$\bar{v}(t) = -3\sin t\,\hat{i} + 2\cos t\,\hat{j}$

$\bar{F}(\bar{r}(t)) = \frac{2}{3}\cdot 3\cos t\,\hat{i} + \left(1+\frac{3}{2}\cdot 2\sin t\right)$

$$\int_C \bar{F}\cdot d\bar{r} = \int_{t=0}^{\pi/2} \bar{F}(\bar{r}(t))\cdot\bar{v}(t)\,dt = \int\left(\cancel{-6\sin t\cos t} + 2\cos t + \cancel{6\cos t\sin t}\right)dt$$

$$= \int_{t=0}^{\pi/2} 2\cos t\,dt$$

$$= 2\,(\sin t)\Big|_0^{\pi/2} \ \boxed{= 2}$$

#6

$$\bar{F}(x,y) = e^{x^2+y}\,\hat{i} + \left(e^{x^2+y} + y^3\right)\hat{j}$$

$$\operatorname{div}\bar{F} = 2x e^{x^2+y} + e^{x^2+y} + 3y^2 = e^{x^2+y}(2x+1) + 3y^2$$

$$\operatorname{curl}\bar{F} = \left(\frac{\partial F^2}{\partial x} - \frac{\partial F^1}{\partial y}\right)\hat{k} = \left(2x e^{(\cdots)} - e^{(\cdots)}\right)\hat{k}$$

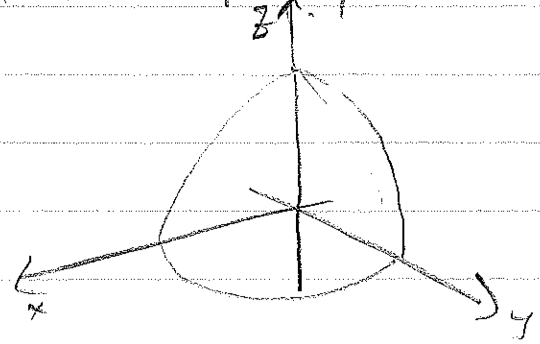
$$= e^{x^2+y}(2x-1)\,\hat{k}$$

Thus, $\operatorname{div}\bar{F} = e^{x^2+y}(2x+1) + 3y^2$ Not irrotational,
$\operatorname{curl}\bar{F} = e^{x^2+y}(2x-1)\,\hat{k}$ Solenoidal, or conservative

#7 Flux of

$\bar{F}(x,y,z) = x\hat{i} + y\hat{j} + (z+1)\hat{k}$ out of

$x^2+y^2+z^2=4$ which is over 1st quadrant



$dS = 4\sin\varphi \, d\varphi \, d\theta$

$0 \le \varphi \le \pi/2$

$0 \le \theta \le \pi/2$

$$\Phi = \iint_S \bar{F} \cdot \hat{N} \, dS$$

Let $G(x,y,z) = x^2+y^2+z^2-4$, so sphere is defined by $G=0$

$\bar{N} = \bar{\nabla} G = 2x\hat{i} + 2y\hat{j} + 2z\hat{k} = 2\bar{\rho}$

$|\bar{N}| = 2\sqrt{x^2+y^2+z^2} = 2 \times 2 = 4$ on the sphere with radius 2.

$\therefore \hat{N} = \bar{\rho}/2$ on the sphere

Note $\bar{F} = \bar{\rho} + 1\hat{k}$, so

$$\iint_S \bar{F} \cdot \hat{N} \, dS = \iint_S \left(\frac{|\bar{\rho}|^2}{2} + \frac{z}{2} \right)\Bigg|_S dS$$

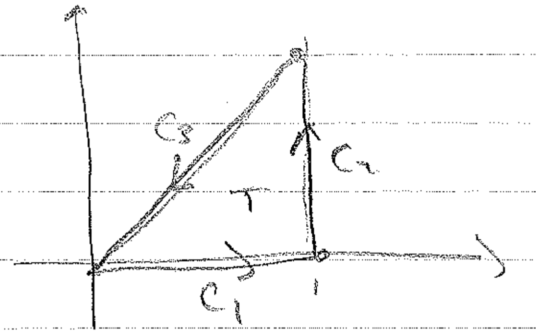
$$= \int_{\varphi=0}^{\pi/2} \int_{\theta=0}^{\pi/2} (2 + \cos\varphi) \, 4\sin\varphi \, d\theta \, d\varphi$$

$$= 2 \times \underbrace{\text{Area (Sphere)}/8}_{4\pi \times 2^2/8} + \underbrace{4 \cdot \frac{\pi}{2} \underbrace{\int_{\varphi=0}^{\pi/2} \cos\varphi \sin\varphi \, d\varphi}_{\int_0^1 u \, du \, = \, \frac{1}{2}}}_{\pi}$$

$\begin{Bmatrix} u = \sin\varphi \\ du = \cos\varphi \, d\varphi \end{Bmatrix}$

$$\boxed{= 5\pi}$$

#8



The curves can be parameterized as:

C_1 $\bar{r}_1(t) = t\hat{i} + 0\hat{j}$, $0 \le t \le 1$

C_2 : $\bar{r}_2(t) = 1\hat{i} + t\hat{j}$, $0 \le t \le 1$

C_3 : $\bar{r}_3(t) = t\hat{i} + t\hat{j}$, t 1 to 0

Green's Theorem: Let C be the counter-clockwise oriented curve which bounds the triangle T, and suppose $\bar{F}(x,y)$ is a smooth vector field on T, then

$$(*) \qquad \oint_C \bar{F}\cdot d\bar{r} = +\iint_T \left(\frac{\partial F^2}{\partial x} - \frac{\partial F^1}{\partial y}\right) dA$$

Proof Consider the r.h.s.

Step 1. $$\iint_T \left(\frac{\partial F^2}{\partial x} - \frac{\partial F^1}{\partial y}\right) dx\,dy = \int_{y=0}^{1}\int_{x=y}^{1} \frac{\partial F^2}{\partial x}\, dx\,dy - \int_{x=0}^{1}\int_{y=0}^{x} \frac{\partial F^1}{\partial y}\, dy\,dx$$

$$= \int_{y=0}^{1} \left(F^2(1,y) - F^2(y,y)\right) dy$$

$$- \int_{x=0}^{1} \left(F^1(x,x) - F^1(x,0)\right) dx$$

$$= \int_{x=0}^{1} F^1(x,0)\, dx + \int_{y=0}^{1} F^2(1,y)\, dy$$

$$+ \int_1^0 F^1(x,x)\, dx + \int_1^0 F^2(y,y)\, dy$$

$$= \int_{t=0}^{1} F^1(t,0)\, dt + \int_{t=0}^{1} F^2(1,t)\, dt$$

$$+ \int_{t=1}^{0} \left(F^1(t,t) + F^2(t,t)\right) dt$$

, changing integration dummy-variable

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Step 2. Now, let the boundary C curves C_1, C_2, C_3 be parameterized as above. The L.H.S. of (*) is then

$$\oint_C \bar{F} \cdot d\bar{r} = \int_{C_1} \bar{F} \cdot d\bar{r} + \int_{C_2} \bar{F} \cdot d\bar{r} + \int_{C_3} \bar{F} \cdot d\bar{r}$$

$$= \int_{t=0}^{1} \bar{F}(\bar{r}_1(t)) \cdot \bar{v}_1(t)\, dt + \int_{t=0}^{1} \bar{F}(\bar{r}_2(t)) \cdot \bar{v}_2(t)\, dt$$

$$+ \int_{t=1}^{0} \bar{F}(\bar{r}_3(t)) \cdot \bar{v}_3(t)\, dt,$$

where $\bar{v}_i = \frac{d\bar{r}_i}{dt}$, $i = 1, 2, 3$.

$$= \int_0^1 F^1(t, 0)\, dt + \int_0^1 F^2(1, t)\, dt$$

$$+ \int_1^0 \left(F^1(t,t) + F^2(t,t) \right) dt$$

By step 1 this is equal to $\iint \left(\frac{\partial F^2}{\partial x} - \frac{\partial F^1}{\partial y} \right) dA$, which was to be shown.

$\square$

#9 $\bar{F} = \bar{\nabla}\phi$ for some ϕ.

a) False

$\bar{\nabla} \times \bar{F} = \bar{\nabla} \times (\bar{\nabla}\phi)$ vanishes identically:

Consider the $\hat{\imath}$ component: $\frac{\partial F^3}{\partial y} - \frac{\partial F^2}{\partial z} = \frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial z}\right) - \frac{\partial}{\partial z}\left(\frac{\partial \phi}{\partial y}\right)$

$= 0$ by equality of mixed partials

The same argument holds for other components showing $\bar{\nabla} \times \bar{F} = 0$.

b) True.

$$\oint_C \bar{F} \cdot d\bar{r} = \oint_C \bar{\nabla}\phi \cdot d\bar{r}$$

$$= \int_{t=a}^{b} \left(\frac{\partial \phi}{\partial x}\frac{dx}{dt} + \frac{\partial \phi}{\partial y}\frac{dy}{dt} + \frac{\partial \phi}{\partial z}\frac{dz}{dt} \right) dt$$, with a parameterization $\bar{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k}$, $a \le t \le b$

$$= \int_{t=a}^{b} \frac{d}{dt}\phi(\bar{r}(t))\, dt$$ by chain rule

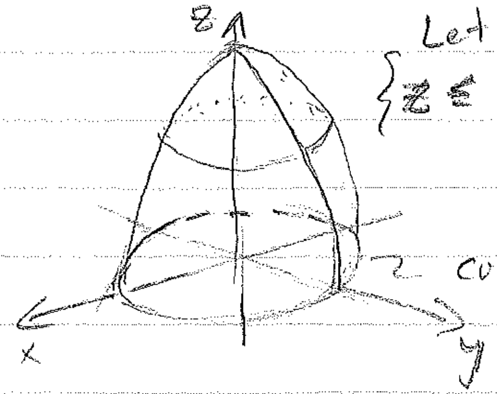
$$= \phi(\bar{r}(b)) - \phi(\bar{r}(a))$$

If C is closed, $\bar{r}(a) = \bar{r}(b) \Rightarrow \oint \bar{F} \cdot d\bar{r} = 0$

#10

By divergence Theorem $\oiint_S \vec{F}\cdot\vec{N}\,dS = \iiint_V \operatorname{div}\vec{F}\,dV$

where S is the closed surface bounding V.



Let V be the paraboloidal region $\{z \le 6 - \frac{3}{2}x^2 - \frac{2}{3}y^2 ,\ z \ge 0\}$

curve in xy-plane is ellipse $1 = \frac{x^2}{4} + \frac{y^2}{9}$

$$\iiint_V \operatorname{div}\vec{F}\,dV = \iiint_V \left(\frac{1}{18}\sqrt{9x^2+4y^2} + \frac{x}{18}\frac{18x}{2\sqrt{9x^2+4y^2}} + \frac{1}{18}\sqrt{9x^2+4y^2} + \frac{y}{18}\frac{8y}{2\sqrt{9x^2+4y^2}} \right) dV$$

$$= \iiint_V \frac{1}{6}\sqrt{9x^2+4y^2}\,dx\,dy\,dz$$

Introduce coordinates $x = 2u,\ y = 3v,\ z = 6w$

The region V becomes

$$\{6w \le 6 - 6u^2 - 6v^2,\ 6w \ge 0\} \Rightarrow \{w \le 1 - u^2 - v^2,\ w \ge 0\}$$

The Jacobian is $\left|\frac{\partial(x,y,z)}{\partial(u,v,w)}\right| = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 6 \end{vmatrix} = 36$

So the integral becomes

$$\iiint_{\tilde{V}} \frac{1}{6}\sqrt{9\cdot 4(u^2+v^2)}\ 36\,du\,dv\,dz = 36\iiint_{\tilde{V}} \sqrt{u^2+v^2}\,du\,dv\,dw$$

$\tilde{V}$ ← region in (u,v,w)-space

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Now introduce cylindrical coordinates in (u,v,w)-space

$$u = r\cos\theta, \quad v = r\sin\theta, \quad \zeta = w$$

The letter zeta to distinguish from original z.

The Jacobian is the usual one

$$\left|\frac{\partial(u,v,w)}{\partial(r,\theta,\zeta)}\right| = r$$

The paraboloidal region is defined by
$\zeta \le 1 - r^2$, $\zeta \ge 0$

So we write the integral in iterated form

$$36 \int_{r=0}^{1}\int_{\zeta=0}^{1-r^2}\int_{\theta=0}^{2\pi} r \, r \, d\theta \, d\zeta \, dr$$

$$= 2\pi \times 36 \int_{r=0}^{1} r^2(1-r^2)\,dr$$

$$= 2\pi \times 36 \left(\frac{1}{3} - \frac{1}{5}\right)$$

$$= 2\pi \times 36 \times \frac{2}{15}$$

$$= 4\pi \, \frac{12}{5} = \frac{48}{5}\pi$$

$\boxed{\text{So the total Flux is } \frac{48}{5}\pi}$