MATEMATIK
Chalmers tekniska högskola

Hjälpmedel: inga, inte ens räknedosa
Datum: 2017–05–05 kl. 08.30 – 09.30

Flervariabelsanalys, Salsdugga 2

Övningsdugga

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Personnummer: ....

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SUMMA:	

1. Vektorfältet $\mathbf{F}(x,y) = (\frac{2x}{y}, \frac{1-x^2}{y^2})$ är konservativt. Beräkna dess potential. (1 p)

Lösning: Om $\phi : \mathbb{R}^2 \to \mathbb{R}$ s.a. $\nabla\phi = \mathbb{F}$ så

$$\frac{\partial\phi}{\partial x} = \frac{2x}{y} \implies \phi(x,y) = \frac{x^2}{y} + g(y) \implies$$

$$\implies \frac{1}{y^2} - \frac{x^2}{y} = F_2 = \frac{\partial\phi}{\partial y} = -\frac{x^2}{y^2} + g'(y) \iff$$

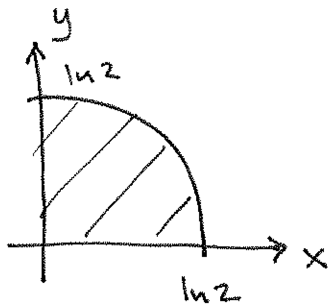
$$\iff g'(y) = \frac{1}{y^2} \implies g(y) = -\frac{1}{y} + C$$

$$\therefore \phi(x,y) = \frac{x^2-1}{y} + C$$

2. Beräkna (1 p)

$$\int_0^{\ln(2)} \left(\int_0^{\sqrt{(\ln(2))^2 - y^2}} e^{\sqrt{x^2+y^2}} dx \right) dy.$$

Lösning: $x = \sqrt{(\ln 2)^2 - y^2} \iff x^2 + y^2 = (\ln 2)^2$



Polära koord.: $0 \leq r \leq \ln 2$
$0 \leq \theta \leq \pi/2$

$$\implies \int_0^{\pi/2} \left(\int_0^{\ln 2} e^r r\,dr \right) d\theta = \frac{\pi}{2} \int_0^{\ln 2} r e^r\,dr =$$

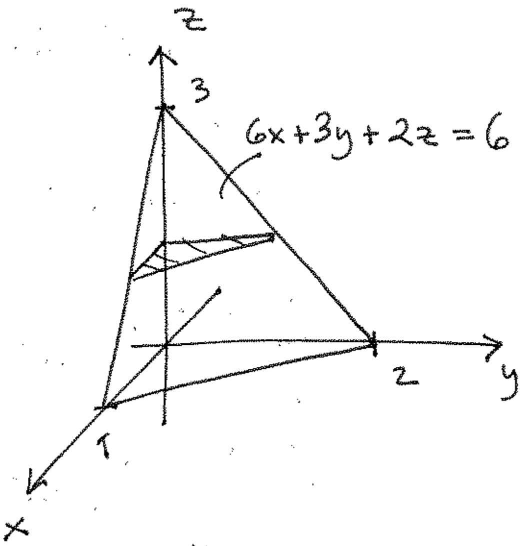
$$= \frac{\pi}{2} \left([re^r]_0^{\ln 2} - \int_0^{\ln 2} e^r\,dr \right) = \frac{\pi}{2} \left(2\ln(2) - [e^r]_0^{\ln 2} \right) =$$

$$= \frac{\pi}{2} \left(\ln(4) - (2-1) \right) = \frac{\pi}{2} (\ln(4) - 1)$$

3. Beräkna $\iiint_T z\, dV$ där T är tetraedern med hörn i punkterna $(0,0,0)$, $(1,0,0)$, $(0,2,0)$ och $(0,0,3)$. (2 p)

Lösning:

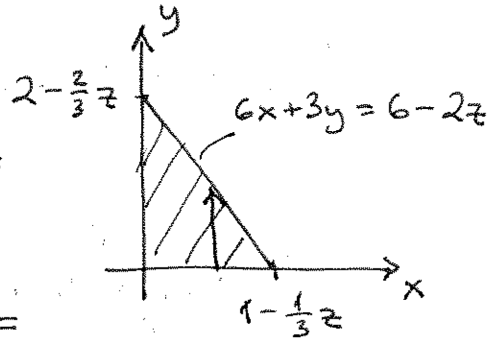
$$\iiint_T z\, dV =$$



$$= \int_0^3 \left(\int_0^{1-\frac{1}{3}z} \left(\int_0^{2-\frac{2}{3}z-2x} z\, dy \right) dx \right) dz =$$

$$= \int_0^3 z \left(\int_0^{1-\frac{z}{3}} \left(2 - \frac{2}{3}z - 2x\right) dx \right) dz =$$

$$= \int_0^3 z \left[2x - \frac{2z}{3}x - x^2 \right]_{x=0}^{x=1-\frac{z}{3}} dz =$$



$$= \int_0^3 z \left(2\left(1-\frac{z}{3}\right) - \frac{2z}{3}\left(1-\frac{z}{3}\right) - \left(1-\frac{z}{3}\right)^2 \right) dz =$$

$$= \int_0^3 z \left(2 - \frac{2z}{3} - \cancel{\frac{2z}{3}} + \frac{2z^2}{9} - 1 + \cancel{\frac{2z}{3}} - \frac{z^2}{9} \right) dz =$$

$$= \int_0^3 z \left(1 - \frac{2z}{3} + \frac{z^2}{9} \right) dz = \int_0^3 \left(z - \frac{2z^2}{3} + \frac{z^3}{9} \right) dz =$$

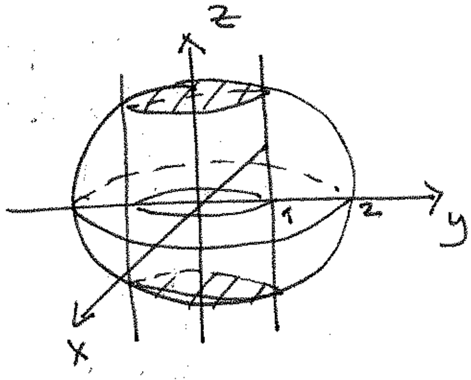
$$= \left[\frac{z^2}{2} - \frac{2z^3}{9} + \frac{z^4}{4\cdot 9} \right]_0^3 = \frac{9}{2} - \frac{2\cdot 3^3}{3^2} + \frac{3^4}{4\cdot 3^2} =$$

$$= \frac{9}{2} - 6 + \frac{9}{4} = \frac{18}{4} - \frac{24}{4} + \frac{9}{4} = \frac{3}{4}$$

4. Beräkna arean av den del av sfären $x^2 + y^2 + z^2 = 4$ som ligger innanför cylindern $x^2 + y^2 = 1$. (2 p)

Lösning: Två lika stora delar.

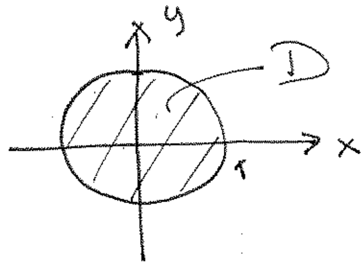
Beräknar då $z > 0$:



$$z = f(x,y) = \sqrt{4 - x^2 - y^2}$$

$$\frac{\partial f}{\partial x} = -\frac{x}{\sqrt{4 - x^2 - y^2}}$$

$$\frac{\partial f}{\partial y} = -\frac{y}{\sqrt{4 - x^2 - y^2}}$$



$$\text{Ytarea} = 2 \iint_D \sqrt{1 + \left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}\, dxdy$$

$$= 2 \iint_D \sqrt{1 + \frac{x^2 + y^2}{4 - x^2 - y^2}}\, dxdy =$$

$$= 2 \iint_D \frac{2}{\sqrt{4 - (x^2 + y^2)}}\, dxdy \overset{\text{pol. koord.}}{=} 4 \int_0^{2\pi} \left(\int_0^1 \frac{r}{\sqrt{4 - r^2}}\, dr \right) d\theta =$$

$$= \left\{ \begin{array}{l} u = 4 - r^2, \quad r = 0 \Leftrightarrow u = 4 \\ du = -2r\,dr, \quad r = 1 \Leftrightarrow u = 3 \end{array} \right\} = 8\pi \int_4^3 \frac{1}{\sqrt{u}} \cdot \frac{du}{-2} =$$

$$= 4\pi \int_3^4 u^{-1/2}\, du = 4\pi \left[\frac{u^{1/2}}{1/2} \right]_3^4 = 8\pi (2 - \sqrt{3}) \text{ a.e.}$$