OPTIONS AND MATHEMATICS (CTH[mve095] , GU[MMA700])
ASSIGNMENTS 2010

Please hand in solutions of at most 5 out of the following 6 problems at the latest Friday, April 16 at 13^{45}. Each problem gives at most $0.5/5 = 0.1$ credit points on the final examination in May 2010.

The MATLAB figures in Problems 2 and 6 must be simple to understand and with defined label and axes. No parts of the MATLAB figures are allowed to be written by hand. Good luck!

Problem 1 Suppose $K > 0$.Find a portfolio consisting of European calls and puts with termination date T such that the value of the portfolio at time T equals

$$Y = \max(0, \mid S(T) - K \mid -\frac{1}{2}K).$$

Problem 2 (MATLAB; title('Problem 1'), legend('Bull spread')) A European derivative pays the amount $g(S(T))$ at time of maturity T, where $g(s) = (s-2)^+ - (s-4)^+$, $0 \leq s \leq 6$. Plot the graph of the payoff function $y = g(s)$.

Problem 3 Let $t < T$ and $N \in \mathbf{N}_+$. Set $\tau = T - t$, $h = \tau/N$, and $t_n = t + nh$, $n = 0, ..., N$. Furthermore, suppose $S = (S(t))_{t \geq 0}$ is a stock price process. A financial contract has the following description: at each point of time t_{n-1} the holder of the contract gets a forward contract on S with delivery date t_n and at time t_n the holder's saving account adds the amount $S(t_n) - S_{for}^{t_n}(t_{n-1})$ for $n = 1, ..., N$. Prove that the sum of the depositions will grow to the amount $S(T) - S_{for}^{T}(t)$ at time T.

Problem 4 *(Sell to the highest;* the binomial model with $d < 0 < r < u$ and $T = 2$) A contingent claim of European type pays the amount

$$Y = \max(S(0), S(1), S(2)) - S(2)$$

at time of maturity 2. Find the replicating strategy at time 1 when it is known that the stock price has declined in the first period of time.

Problem 5 Below θ_i, $i = 0, 1, ..., n$, are positive real numbers such that $\Sigma_0^n \theta_i = 1$.

(a) Suppose $f : I \to \mathbf{R}$ is convex and $x_0, ..., x_n \in I$, where I is a subinterval of $\mathbf{R}$. Show that

$$f(\Sigma_0^n \theta_i x_i) \leq \Sigma_0^n \theta_i f(x_i).$$

(Hint: Use induction on n.)

(b) Suppose $a_i > 0$, $i = 0, 1, ..., n$. Show that

$$\Pi_0^n a_i^{\theta_i} \leq \Sigma_0^n \theta_i a_i.$$

(Hint: If $f :]0, \infty[\to \mathbf{R}$ is differentiable and f' increasing, then f is convex.)

(c) Suppose $K > 0$ and $t \leq t_0 \leq ...t_n \leq T$. Two European derivatives with maturity date T have the payoffs

$$Y_1 = (\Pi_0^n S(t_i)^{\theta_i} - K)^+$$

and

$$Y_2 = (\Sigma_0^n \theta_i S(t_i) - K)^+,$$

respectively. Show that

$$\Pi_{Y_1}(t) \leq \Pi_{Y_2}(t) \leq c(t, S(t), K, T).$$

Problem 6 (MATLAB; title('Problem 6'), legend('$N(0,\sigma)$ density for various σ')) Plot the graph of the function $z = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{x^2}{2\sigma^2}}$, $-10 \leq x \leq 10$, $1 \leq \sigma \leq 5$.